Maker-Breaker Games: Building a Big Chain in a Poset

Daniel W. Cranston
Virginia Commonwealth University
dcranston@vcu.edu

Joint with Bill Kinnersley, Kevin Milans, Greg Puleo, Douglas West

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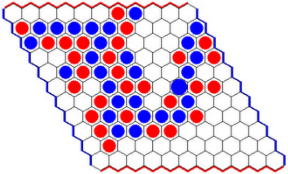

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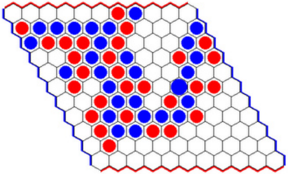

We want to find the threshold where the game switches from a Breaker win to a Maker win.

Subset Lattices

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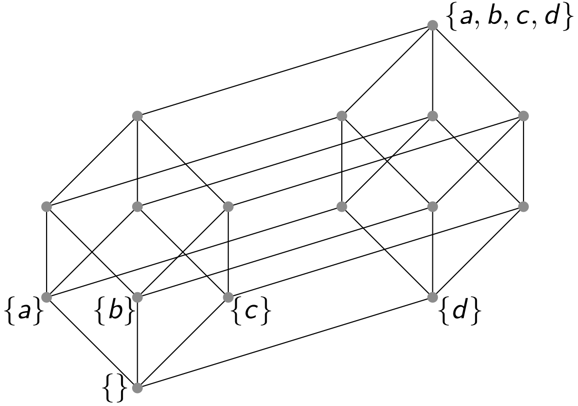

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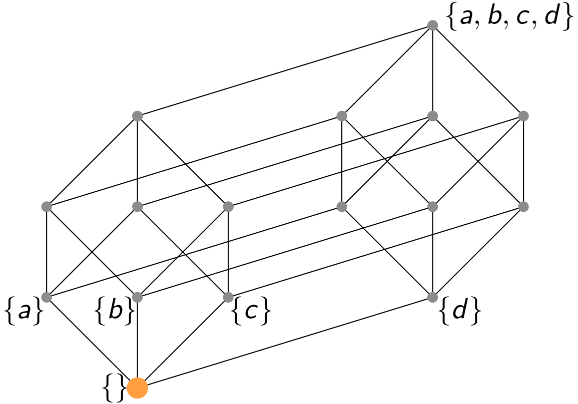

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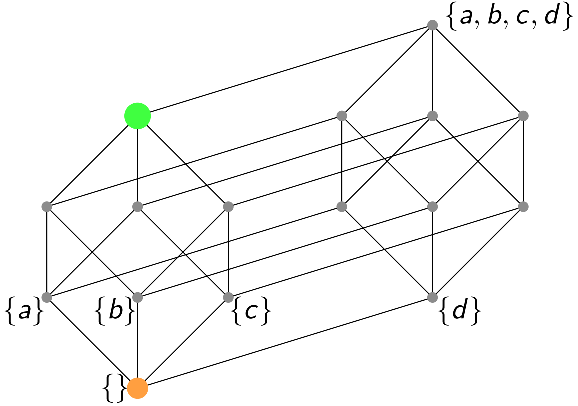

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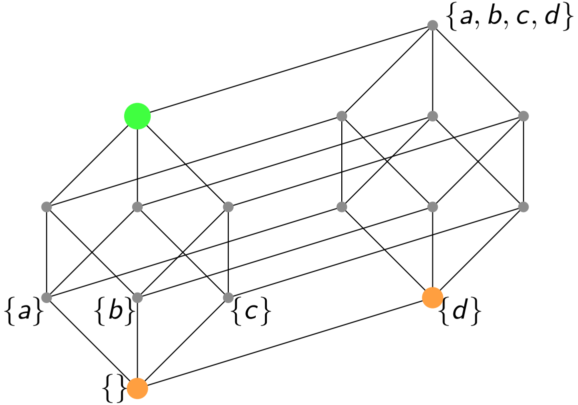

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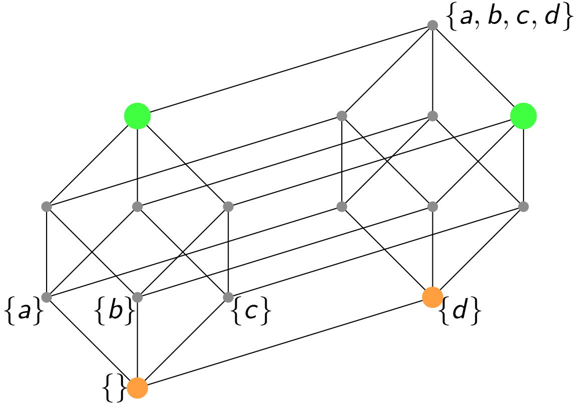

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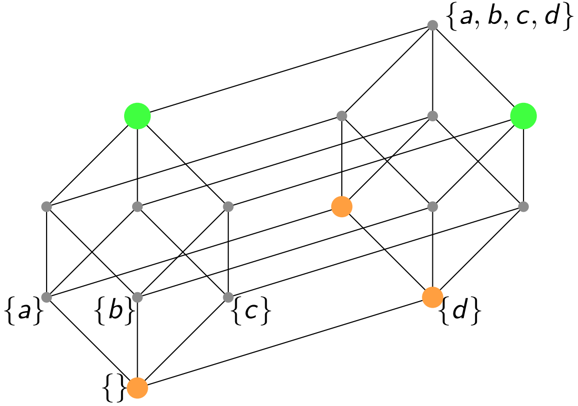

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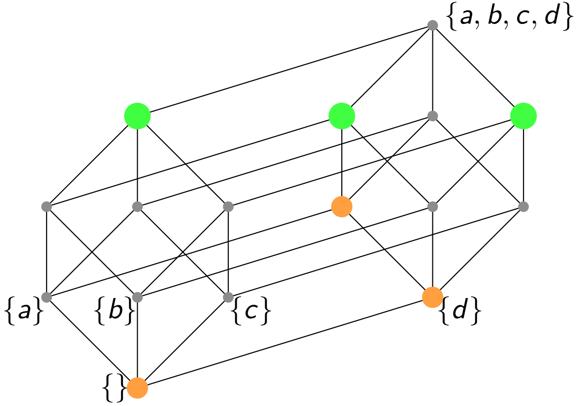

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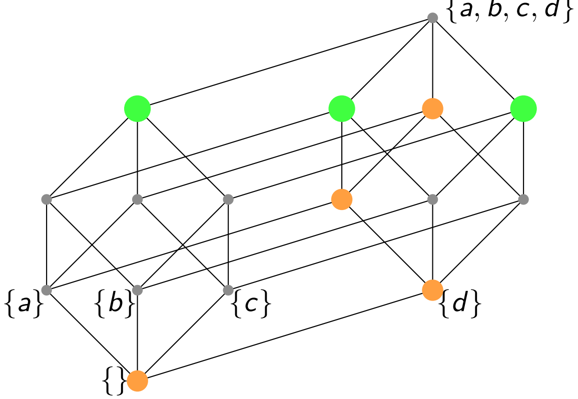

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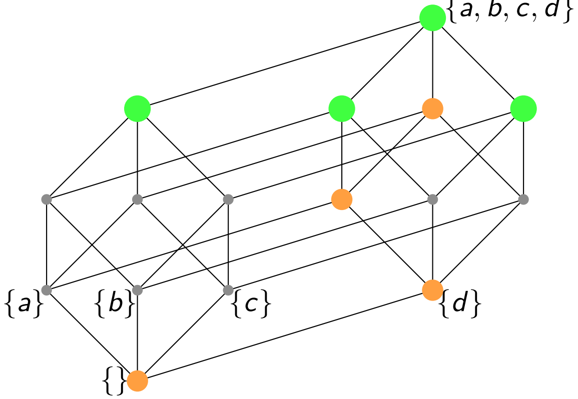

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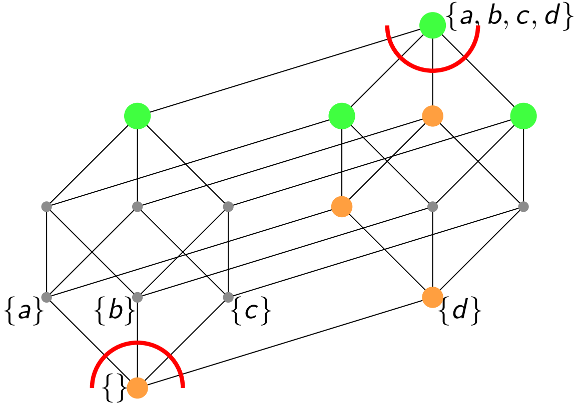

Corollary

In the poset $\widehat{L_n}$, Maker can get a maximum size chain.

Product of Two Chains

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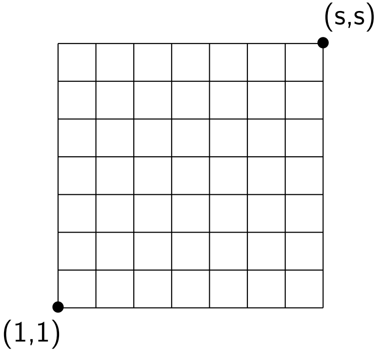

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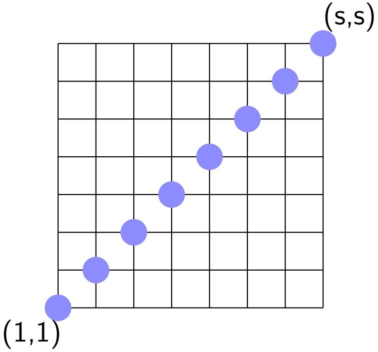

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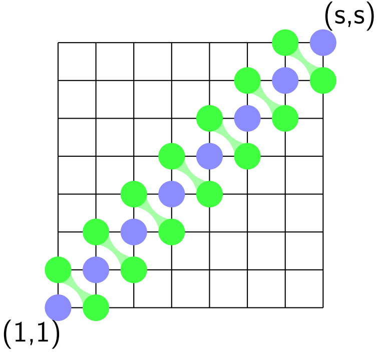

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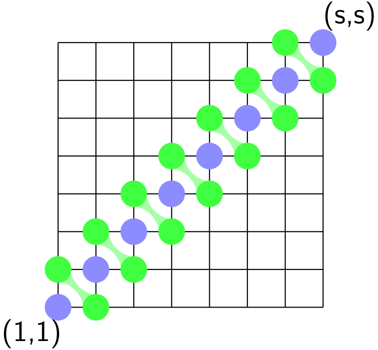

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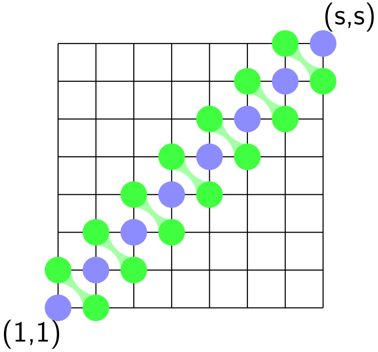

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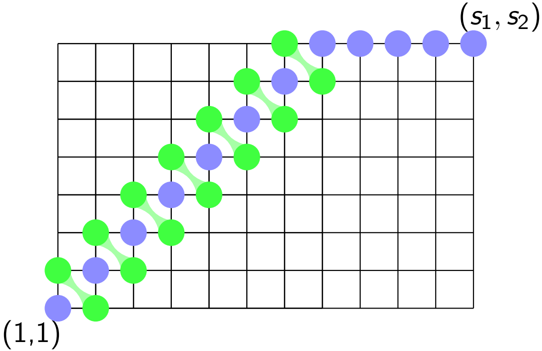

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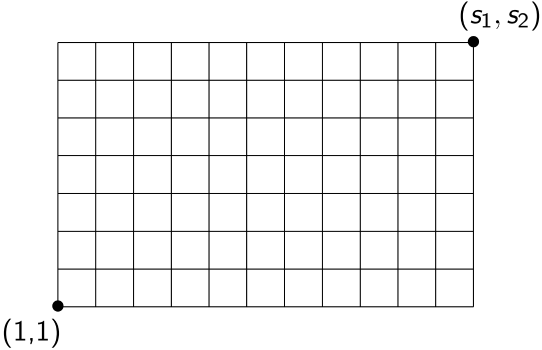

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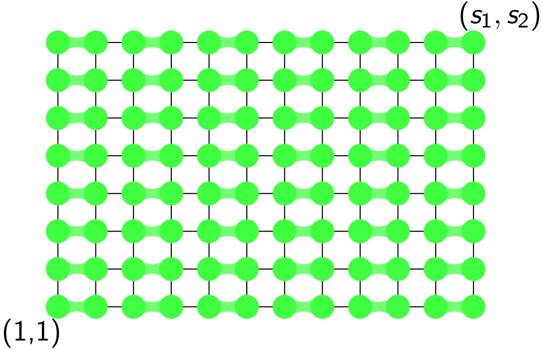

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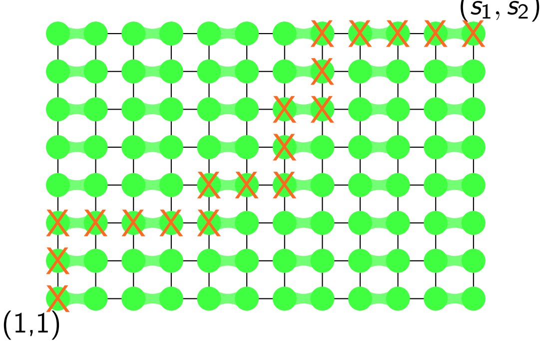

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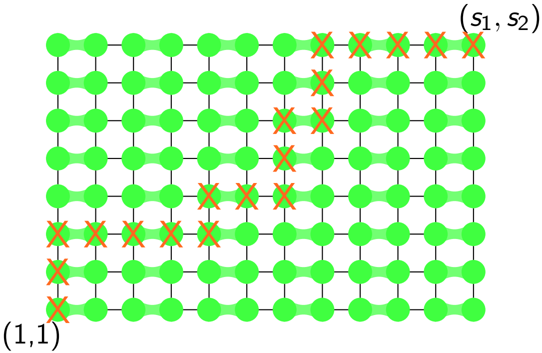

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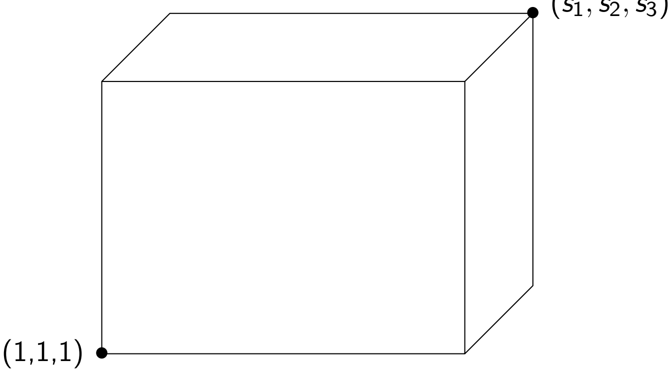

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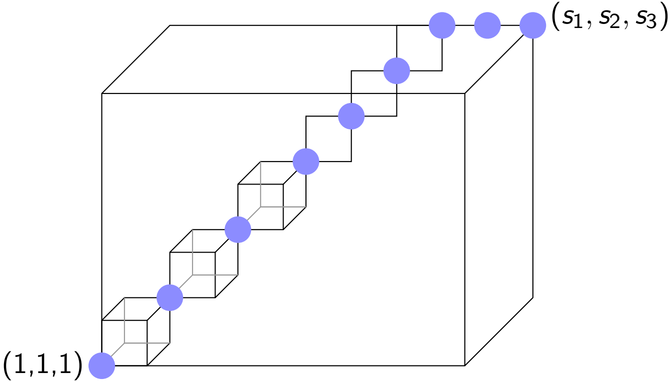

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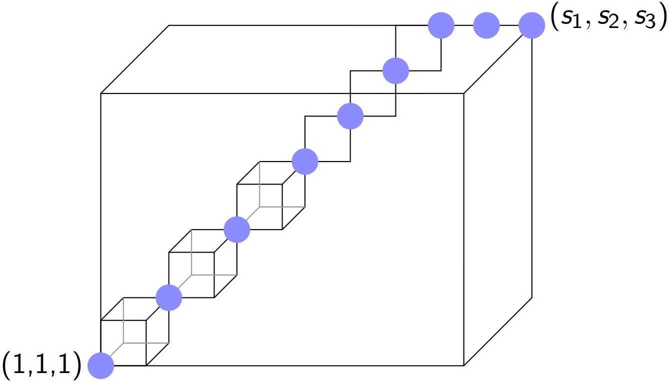

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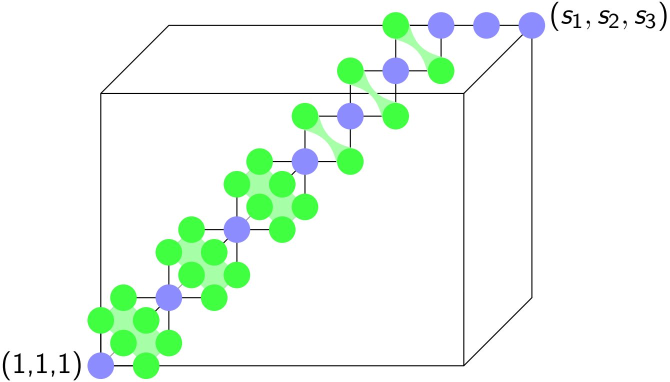

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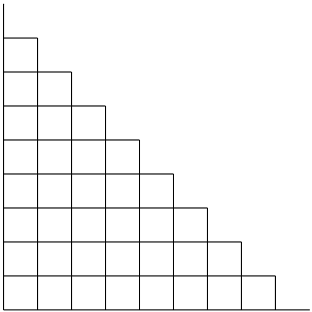

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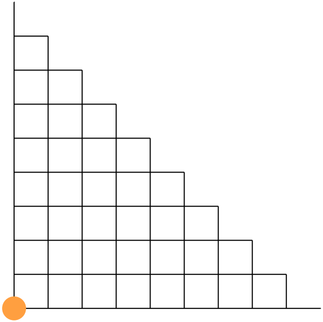

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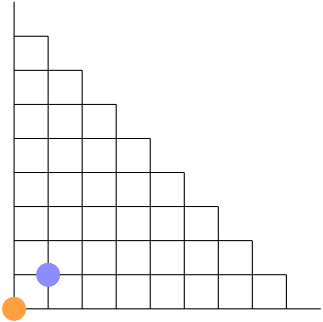

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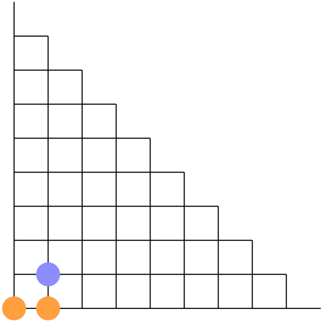

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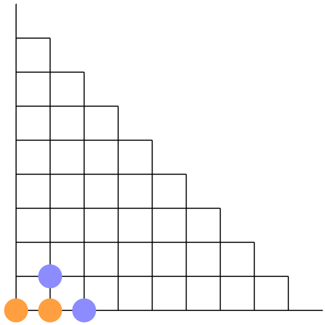

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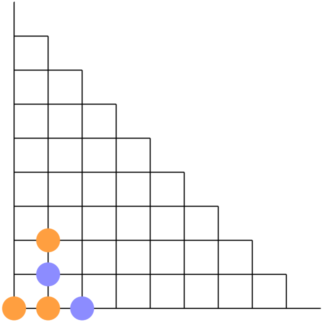

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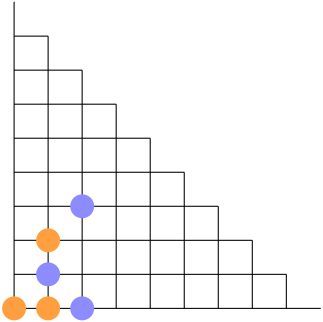

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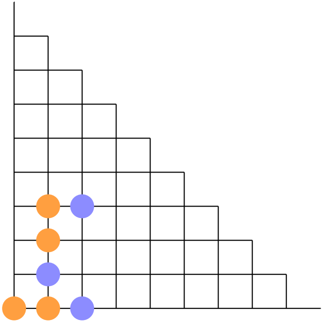

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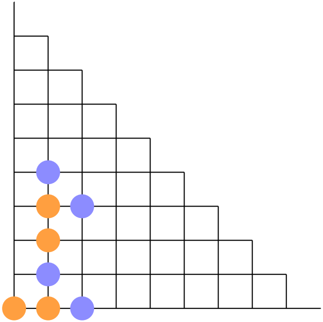

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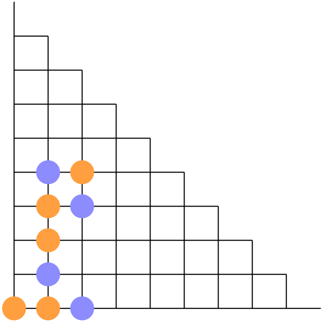

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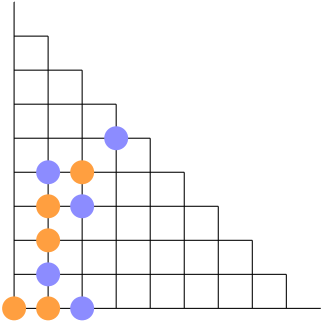

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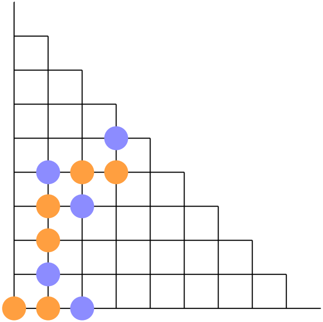

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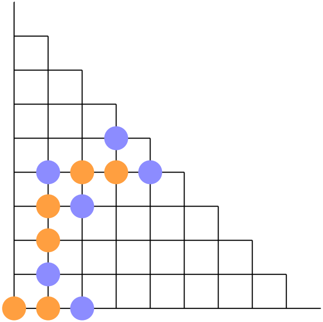

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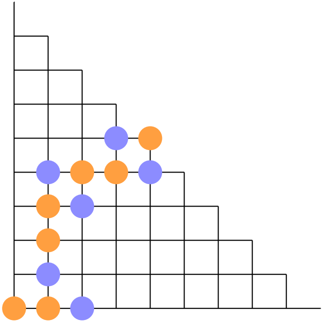

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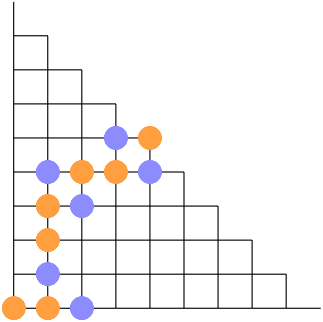

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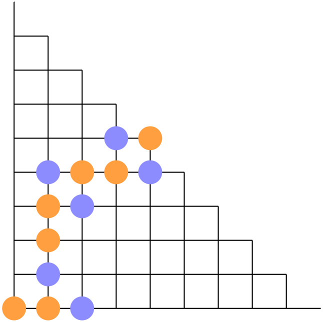

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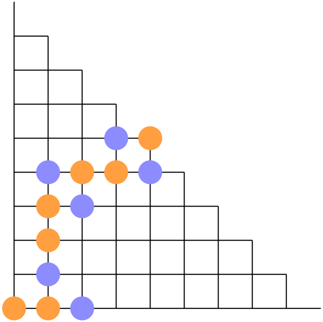

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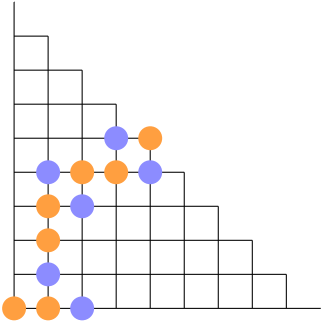

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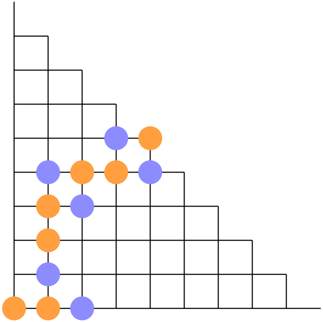

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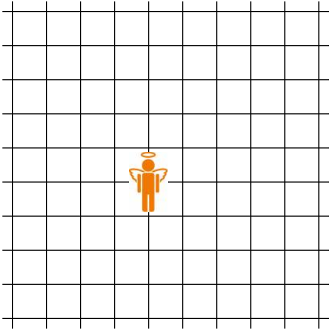

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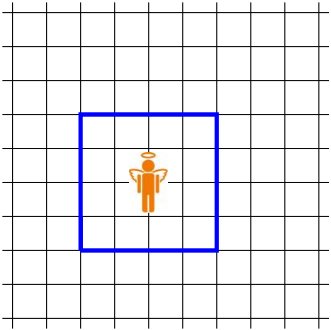

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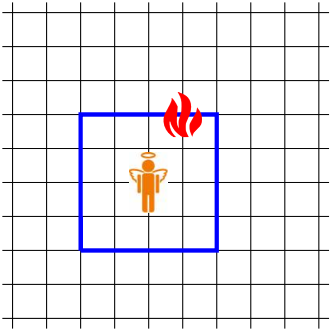

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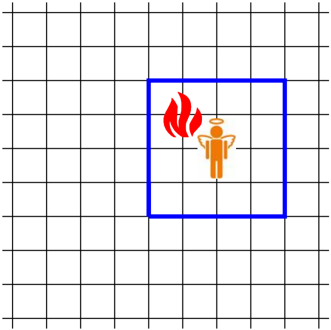

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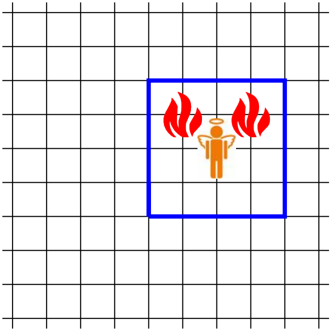

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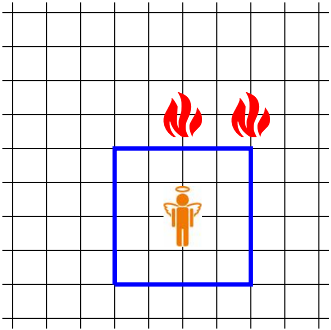

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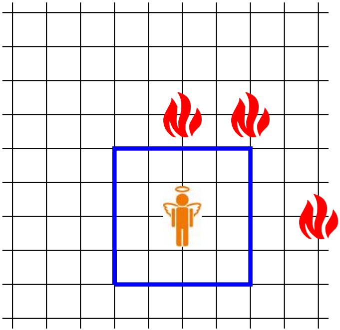

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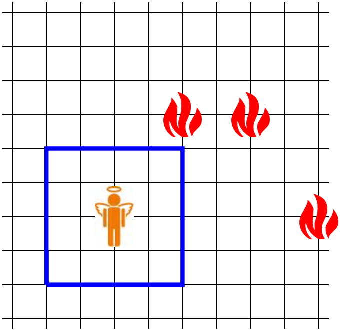

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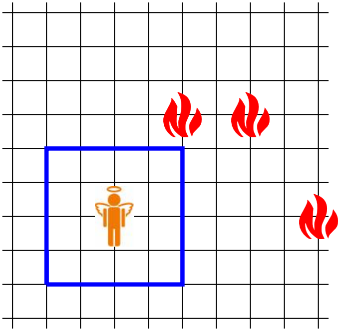

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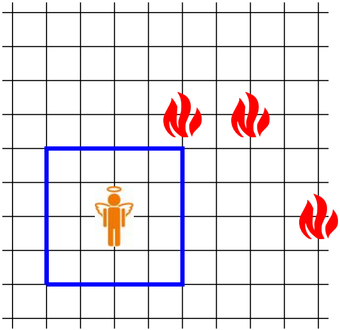

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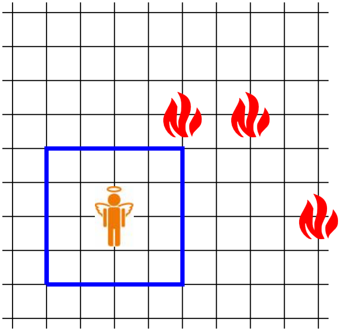

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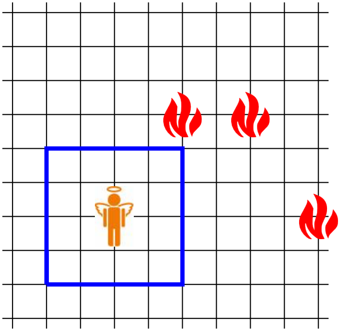

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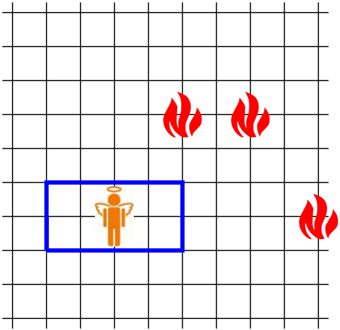

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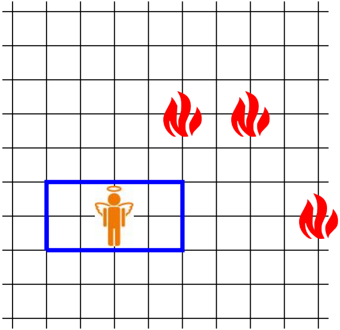

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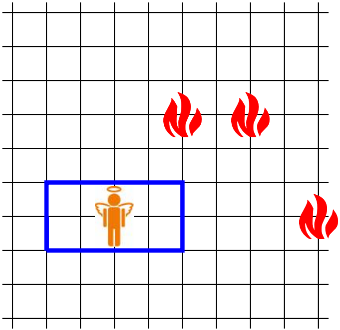

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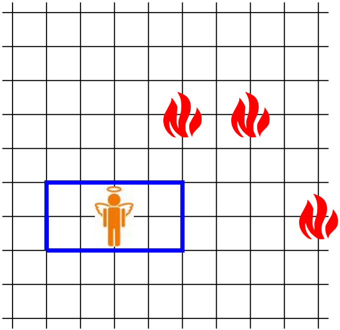

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Conjecture
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