Maker-Breaker Games: 
Building a Big Chain in a Poset

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Maker-Breaker Games (in General)

Maker-Breaker Game:
Two players, Maker and Breaker, alternate turns. On each turn, the player chooses a not-yet-picked element from a base set.
Maker-Breaker Games (in General)

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Two players, **Maker** and **Breaker**, alternate turns. On each turn, the player chooses a not-yet-picked element from a base set. **Maker** tries to collect all the elements in at least one winning subset (i.e. trying to make that subset). **Breaker** tries to stop him.
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We want to find the threshold where the game switches from a Breaker win to a Maker win.
Subset Lattices

Theorem

In the subset lattice $L_n$, Maker $\bullet$ can get a chain of size $n$ that misses only the top element.
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Theorem
In the subset lattice $L_n$, Maker $\blacklozenge$ can get a chain of size $n$ that misses only the top element.

Corollary
In the poset $\hat{L}_n$, Maker can get a maximum size chain.
Theorem

If $P$ is the product of two chains, each of size $s$, then Maker can build a chain in $P$ of size at least $\lceil \frac{3}{2}s \rceil - 1$. 
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Maker’s Strategy
If Breaker plays a green, then Maker plays its pair. Otherwise, Maker plays a blue, if he can.
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**Maker’s Strategy**

If Breaker plays a green, then Maker plays its pair. Otherwise, Maker plays a blue, if he can. Thus: $\left\lceil \frac{1}{2}s \right\rceil + (s - 1) = \left\lceil \frac{3}{2}s \right\rceil - 1$. 
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Theorem

If $P$ is the product of two chains, of sizes $s_1, s_2$ with $s_1 \geq s_2$, then Maker can build a chain in $P$ of size at least $\lceil \frac{1}{2} s_1 \rceil + s_2 - 1$.

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$$(s_1, s_2)$$

$$(1,1)$$
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Breaker’s Strategy
Pair elements “length-wise”. Whatever element Maker plays, Breaker plays its pair.
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Pair elements “length-wise”. Whatever element Maker plays, Breaker plays its pair. So: $(s_1 + s_2 - 1) - \left\lfloor \frac{1}{2} s_1 \right\rfloor = \left\lceil \frac{1}{2} s_1 \right\rceil + s_2 - 1$. 
Product of $d$ Chains

**Theorem**

*If $P$ is the product of $d$ chains, with sizes $s_1 \geq \cdots \geq s_d$, then a maximum chain in $P$ has size $S = \sum s_i - (d - 1)$. Maker can build a chain in $P$ of size at least $S - \left\lfloor \frac{1}{2}s_1 \right\rfloor$.***
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![Diagram](image-url)
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*Like before, but Maker’s old green pairs now become green $\hat{L}_k$s.*
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Walker-Blocker on the Wedge

The wedge $W_k^d$ is $\{(x_1, \ldots, x_d) | x_i \geq 0 \text{ and } \sum x_i < k\}$. 
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\begin{align*}
\ell_W + \ell_B &= k \\
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\ell_B &\leq \frac{1}{3} k
\end{align*}

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Angel-Devil game

**Angel-Devil Game**

Angel: Move from \((x, y)\) to \((x_1, y_1)\) if \(|x - x_1| \leq 2\) and \(|y - y_1| \leq 2\).

Devil: Burn one point \((x, y)\).

*Question* [Conway 1982]: Can the angel move forever?

*Answer* [M´ath´e, Kloster 2006]: Yes!

*Theorem* In the wedge, Walker can get all levels.

*Question*: What about \(W_3^k\) through \(W_{13}^k\)?

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**Theorem**

*In the wedge* \(W_{24}^k\), *Walker can get all levels.*
Angel-Devil game

**Angel-Devil Game**

Angel: Move from \((x, y)\) to \((x_1, y_1)\) if \(|x - x_1| \leq 2\) and \(|y - y_1| \leq 2\).

Devil: Burn one point \((x, y)\).

Question [Conway 1982]:
Can the angel move forever?

Answer [Máthé, Kloster 2006]: Yes!

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Theorem
In the wedge \(W_{14}^{14}\), Walker can get all levels.
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**Theorem**

*In the wedge \(W_k^{14}\), Walker can get all levels.*

Question: What about \(W_k^3\) through \(W_k^{13}\)?
Angel-Devil game

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**Angel:** Move from \((x, y)\) to \((x_1, y_1)\) if \(|x - x_1| \leq 2\) and \(|y - y_1| \leq 2\).

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**Question [Conway 1982]:** Can the angel move forever?

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**Theorem**

*In the wedge \(W_k^{14}\), Walker can get all levels.*

**Question:** What about \(W_k^3\) through \(W_k^{13}\)?

**Conjecture**

*In the wedge \(W_k^3\), Walker can get all the levels.*