In the 70s, Knopfmacher invented Abstract Analytic Number Theory as a framework for generalizing the Prime Number Theorem to arithmetical semigroups i.e. unique factorization commutative monoids with a norm map. In this talk we introduce arithmetical semirings which combine an additive and a multiplicative semigroup structure.

We then prove some basic results suggesting a close relationship between the asymptotic distribution of additive primes and the asymptotic distribution of multiplicative primes. As an application, we show that when fed with Wright’s asymptotic formulas for the number of (unlabelled) connected graphs, our machinery spits out some simple estimates on the asymptotic distribution of cartesian prime graphs.