The Clar number was proposed in chemical graph theory as an index for graphs representing benzenoid molecules. Its definition can be readily extended to plane bipartite graphs and it is computed as the optimal value of a particular maximization problem over the set of perfect matchings of the graph. The Fries number is another similar index. Abeledo and Atkinson showed that the Clar and Fries optimization problems can be solved as linear programs since the coefficient matrices of the equality constraints are unimodular (though they may fail to be totally unimodular).

We give a new proof of the unimodularity of these matrices based on polyhedral projection arguments and derive new formulations for these two problems which have totally unimodular coefficient matrices. We also present strong combinatorial duality results.