A diagram is a digraph $D$ whose vertices are objects in a category, and each arc is labeled with a morphism from its initial to terminal vertex. Thus each directed path induces a morphism from its initial to terminal vertex, by composition. We say the diagram commutes if for each pair $X, Y$ of vertices, any two directed $XY$-paths induce the same morphism.

We focus on the problem of determining if a given diagram commutes. If $D$ is an $n$-vertex tournament, for example, a brute-force attack involves checking more than $O(n!)$ pairs of paths. Must we check them all?

We will see that commutativity of any $D$ follows from commutativity on a small set of $|E(D)| - |V(D)| + 1 \leq O(n^2)$ pairs of paths.

But there is a twist: The “obvious” solution does not work.

(Joint work with Paul Kainen).

For the DM seminar schedule, see:

http://www.people.vcu.edu/~dcranston/DM-seminar