Discovery

Graph the two lines represented by the system of equations.

$$\begin{aligned} x - 2y &= 1 \\ -2x + 3y &= -3 \end{aligned}$$

You can use Gaussian elimination to solve this system as follows.

$$\begin{aligned} x - 2y &= 1 \\ -1y &= -1 \end{aligned} \qquad \begin{aligned} x - 2y &= 1 \\ y &= 1 \end{aligned} \qquad \begin{aligned} x &= 3 \\ y &= 1 \end{aligned}$$

Graph the system of equations you obtain at each step of this process. What do you observe about the lines? You are asked to repeat this graphical analysis for other systems in Exercises 91 and 92.

SECTION 1.1 Exercises

In Exercises 1–6, determine whether the equation is linear in the variables x and y.

1. $2x - 3y = 4$

2. $3x - 4xy = 0$

3. $\frac{3}{y} + \frac{2}{x} - 1 = 0$

4. $x^2 + y^2 = 4$

5. $2 \sin x - y = 14$

6. $(\sin 2)x - y = 14$

In Exercises 7–10, find a parametric representation of the solution set of the linear equation.

7. $2x - 4y = 0$

8. $3x - \frac{1}{2}y = 9$

9. $x + y + z = 1$

10. $13x_1 - 26x_2 + 39x_3 = 13$

In Exercises 11–16, use back-substitution to solve the system.

11. $\begin{aligned} x_1 - x_2 &= 2 \\ x_2 &= 3 \end{aligned}$

12. $\begin{aligned} 2x_1 - 4x_2 &= 6 \\ 3x_2 &= 9 \end{aligned}$

13. $\begin{aligned} -x + y - z &= 0 \\ 2y + z &= 3 \\ \tfrac{1}{2}z &= 0 \end{aligned}$

14. $\begin{aligned} x - y \qquad &= 4 \\ 2y + z &= 6 \\ 3z &= 6 \end{aligned}$

15. $\begin{aligned} 5x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + x_2 \qquad &= 0 \end{aligned}$

16. $\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_2 \qquad &= 0 \end{aligned}$

In Exercises 17–30, graph each system of equations as a pair of lines in the xy-plane. Solve each system and interpret your answer.

17. $\begin{aligned} 2x + y &= 4 \\ x - y &= 2 \end{aligned}$

18. $\begin{aligned} x + 3y &= 2 \\ -x + 2y &= 3 \end{aligned}$

19. $\begin{aligned} x - y &= 1 \\ -2x + 2y &= 5 \end{aligned}$

20. $\begin{aligned} \tfrac{1}{2}x - \tfrac{1}{3}y &= 1 \\ -2x + \tfrac{4}{3}y &= -4 \end{aligned}$

21. $\begin{aligned} 3x - 5y &= 7 \\ 2x + y &= 9 \end{aligned}$

22. $\begin{aligned} -x + 3y &= 17 \\ 4x + 3y &= 7 \end{aligned}$

23. $\begin{aligned} 2x - y &= 5 \\ 5x - y &= 11 \end{aligned}$

24. $\begin{aligned} x - 5y &= 21 \\ 6x + 5y &= 21 \end{aligned}$

25. $\begin{aligned} \frac{x+3}{4} + \frac{y-1}{3} &= 1 \\ 2x - y &= 12 \end{aligned}$

26. $\begin{aligned} \frac{x-1}{2} + \frac{y+2}{3} &= 4 \\ x - 2y &= 5 \end{aligned}$

27. $\begin{aligned} 0.05x - 0.03y &= 0.07 \\ 0.07x + 0.02y &= 0.16 \end{aligned}$

28. $\begin{aligned} 0.2x - 0.5y &= -27.8 \\ 0.3x + 0.4y &= 68.7 \end{aligned}$

29. $\begin{aligned} \frac{x}{4} + \frac{y}{6} &= 1 \\ x - y &= 3 \end{aligned}$

30. $\begin{aligned} \frac{2}{3}x + \frac{1}{6}y &= \frac{2}{3} \\ 4x + y &= 4 \end{aligned}$

In Exercises 31–36, complete the following set of tasks for each system of equations.

(a) Use a graphing utility to graph the equations in the system.

(b) Use the graphs to determine whether the system is consistent or inconsistent.

(c) If the system is consistent, approximate the solution.

(d) Solve the system algebraically.

(e) Compare the solution in part (d) with the approximation in part (c). What can you conclude?

31. $\begin{aligned} -3x - y &= 3 \\ 6x + 2y &= 1 \end{aligned}$

32. $\begin{aligned} 4x - 5y &= 3 \\ -8x + 10y &= 14 \end{aligned}$

33. $\begin{aligned} 2x - 8y &= 3 \\ \tfrac{1}{2}x + y &= 0 \end{aligned}$

34. $\begin{aligned} 9x - 4y &= 5 \\ \tfrac{1}{2}x + \tfrac{1}{3}y &= 0 \end{aligned}$

35. $\begin{aligned} 4x - 8y &= 9 \\ 0.8x - 1.6y &= 1.8 \end{aligned}$

36. $\begin{aligned} -5.3x + 2.1y &= 1.25 \\ 15.9x - 6.3y &= -3.75 \end{aligned}$

The symbol indicates an exercise in which you are instructed to use a graphing utility or a symbolic computer software program.

In Exercises 37–56, solve the system of linear equations.

37. $$\begin{aligned} x_1 - x_2 &= 0 \\ 3x_1 - 2x_2 &= -1 \end{aligned}$$

38. $$\begin{aligned} 3x + 2y &= 2 \\ 6x + 4y &= 14 \end{aligned}$$

39. $$\begin{aligned} 2u + v &= 120 \\ u + 2v &= 120 \end{aligned}$$

40. $$\begin{aligned} x_1 - 2x_2 &= 0 \\ 6x_1 + 2x_2 &= 0 \end{aligned}$$

41. $$\begin{aligned} 9x - 3y &= -1 \\ \tfrac{1}{5}x + \tfrac{2}{5}y &= -\tfrac{1}{3} \end{aligned}$$

42. $$\begin{aligned} \tfrac{2}{3}x_1 + \tfrac{1}{6}x_2 &= 0 \\ 4x_1 + x_2 &= 0 \end{aligned}$$

43. $$\begin{aligned} \frac{x-1}{2} + \frac{y+2}{3} &= 4 \\ x - 2y &= 5 \end{aligned}$$

44. $$\begin{aligned} \frac{x_1+3}{4} + \frac{x_2-1}{3} &= 1 \\ 2x_1 - x_2 &= 12 \end{aligned}$$

45. $$\begin{aligned} 0.02x_1 - 0.05x_2 &= -0.19 \\ 0.03x_1 + 0.04x_2 &= 0.52 \end{aligned}$$

46. $$\begin{aligned} 0.05x_1 - 0.03x_2 &= 0.21 \\ 0.07x_1 + 0.02x_2 &= 0.17 \end{aligned}$$

47. $$\begin{aligned} x + y + z &= 6 \\ 2x - y + z &= 3 \\ 3x \quad - z &= 0 \end{aligned}$$

48. $$\begin{aligned} x + y + z &= 2 \\ -x + 3y + 2z &= 8 \\ 4x + y \quad &= 4 \end{aligned}$$

49. $$\begin{aligned} 3x_1 - 2x_2 + 4x_3 &= 1 \\ x_1 + x_2 - 2x_3 &= 3 \\ 2x_1 - 3x_2 + 6x_3 &= 8 \end{aligned}$$

50. $$\begin{aligned} 5x_1 - 3x_2 + 2x_3 &= 3 \\ 2x_1 + 4x_2 - x_3 &= 7 \\ x_1 - 11x_2 + 4x_3 &= 3 \end{aligned}$$

51. $$\begin{aligned} 2x_1 + x_2 - 3x_3 &= 4 \\ 4x_1 \quad + 2x_3 &= 10 \\ -2x_1 + 3x_2 - 13x_3 &= -8 \end{aligned}$$

52. $$\begin{aligned} x_1 \quad + 4x_3 &= 13 \\ 4x_1 - 2x_2 + x_3 &= 7 \\ 2x_1 - 2x_2 - 7x_3 &= -19 \end{aligned}$$

53. $$\begin{aligned} x - 3y + 2z &= 18 \\ 5x - 15y + 10z &= 18 \end{aligned}$$

54. $$\begin{aligned} x_1 - 2x_2 + 5x_3 &= 2 \\ 3x_1 + 2x_2 - x_3 &= -2 \end{aligned}$$

55. $$\begin{aligned} x + y + z + w &= 6 \\ 2x + 3y \quad - w &= 0 \\ -3x + 4y + z + 2w &= 4 \\ x + 2y - z + w &= 0 \end{aligned}$$

56. $$\begin{aligned} x_1 \quad + 3x_4 &= 4 \\ 2x_2 - x_3 - x_4 &= 0 \\ 3x_2 \quad - 2x_4 &= 1 \\ 2x_1 - x_2 + 4x_3 \quad &= 5 \end{aligned}$$

In Exercises 57–64, use a computer software program or graphing utility to solve the system of linear equations.

57. $$\begin{aligned} x_1 + 0.5x_2 + 0.33x_3 + 0.25x_4 &= 1.1 \\ 0.5x_1 + 0.33x_2 + 0.25x_3 + 0.21x_4 &= 1.2 \\ 0.33x_1 + 0.25x_2 + 0.2x_3 + 0.17x_4 &= 1.3 \\ 0.25x_1 + 0.2x_2 + 0.17x_3 + 0.14x_4 &= 1.4 \end{aligned}$$

58. $$\begin{aligned} 0.1x - 2.5y + 1.2z - 0.75w &= 108 \\ 2.4x + 1.5y - 1.8z + 0.25w &= -81 \\ 0.4x - 3.2y + 1.6z - 1.4w &= 148.8 \\ 1.6x + 1.2y - 3.2z + 0.6w &= -143.2 \end{aligned}$$

59. $$\begin{aligned} 123.5x + 61.3y - 32.4z &= -262.74 \\ 54.7x - 45.6y + 98.2z &= 197.4 \\ 42.4x - 89.3y + 12.9z &= 33.66 \end{aligned}$$

60. $$\begin{aligned} 120.2x + 62.4y - 36.5z &= 258.64 \\ 56.8x - 42.8y + 27.3z &= -71.44 \\ 88.1x + 72.5y - 28.5z &= 225.88 \end{aligned}$$

61. $$\begin{aligned} \tfrac{1}{2}x_1 - \tfrac{3}{7}x_2 + \tfrac{2}{9}x_3 &= \tfrac{349}{630} \\ \tfrac{2}{3}x_1 + \tfrac{4}{9}x_2 - \tfrac{2}{5}x_3 &= -\tfrac{19}{45} \\ \tfrac{4}{5}x_1 - \tfrac{1}{8}x_2 + \tfrac{4}{3}x_3 &= \tfrac{139}{150} \end{aligned}$$

62. $$\begin{aligned} \tfrac{1}{4}x_1 - \tfrac{3}{5}x_2 + \tfrac{1}{3}x_3 &= \tfrac{43}{60} \\ \tfrac{2}{5}x_1 + \tfrac{1}{4}x_2 - \tfrac{5}{6}x_3 &= -\tfrac{331}{600} \\ \tfrac{3}{4}x_1 - \tfrac{2}{5}x_2 + \tfrac{1}{5}x_3 &= \tfrac{81}{100} \end{aligned}$$

63. $$\begin{aligned} \tfrac{1}{8}x - \tfrac{1}{7}y + \tfrac{1}{6}z - \tfrac{1}{5}w &= 1 \\ \tfrac{1}{7}x + \tfrac{1}{6}y - \tfrac{1}{5}z + \tfrac{1}{4}w &= 1 \\ \tfrac{1}{6}x - \tfrac{1}{5}y + \tfrac{1}{4}z - \tfrac{1}{3}w &= 1 \\ \tfrac{1}{5}x + \tfrac{1}{4}y - \tfrac{1}{3}z + \tfrac{1}{2}w &= 1 \end{aligned}$$

64. $$\begin{aligned} \tfrac{1}{8}x + \tfrac{1}{7}y - \tfrac{1}{6}z + \tfrac{1}{5}w &= 1 \\ \tfrac{1}{7}x - \tfrac{1}{6}y + \tfrac{1}{5}z - \tfrac{1}{4}w &= 1 \\ \tfrac{1}{6}x + \tfrac{1}{5}y - \tfrac{1}{4}z + \tfrac{1}{3}w &= 1 \\ \tfrac{1}{5}x - \tfrac{1}{4}y + \tfrac{1}{3}z - \tfrac{1}{2}w &= 1 \end{aligned}$$

In Exercises 65–68, state why each system of equations must ha at least one solution. Then solve the system and determine if it h exactly one solution or an infinite number of solutions.

65. $$\begin{aligned} 4x + 3y + 17z &= 0 \\ 5x + 4y + 22z &= 0 \\ 4x + 2y + 19z &= 0 \end{aligned}$$

66. $$\begin{aligned} 2x + 3y \quad &= 0 \\ 4x + 3y - z &= 0 \\ 8x + 3y + 3z &= 0 \end{aligned}$$

67. $$\begin{aligned} 5x + 5y - z &= 0 \\ 10x + 5y + 2z &= 0 \\ 5x + 15y - 9z &= 0 \end{aligned}$$

68. $$\begin{aligned} 12x + 5y + z &= 0 \\ 12x + 4y - z &= 0 \end{aligned}$$

True or False? In Exercises 69 and 70, determine whether e statement is true or false. If a statement is true, give a reason or an appropriate statement from the text. If a statement is fa provide an example that shows the statement is not true in all c or cite an appropriate statement from the text.

69. (a) A system of one linear equation in two variables is alv consistent.

(b) A system of two linear equations in three variables is alv consistent.

(c) If a linear system is consistent, then it has an inf number of solutions.

The symbol CL indicates that electronic data sets for these exercises are available at *college.cengage.com/pic/larsonELA6e*. These data sets are compatible with each of the following technologies: MATLAB, *Mathematica*, Maple, Derive, TI-83/TI-83 Plus, TI-84/TI-84 Plus, TI-86, TI-89, TI-92, and TI-92 Plus.

SECTION 1.2 Exercises

In Exercises 1–8, determine the size of the matrix.

1. $\begin{bmatrix} 1 & 2 & -4 \\ 3 & -4 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 2 & -1 & 4 & 2 \\ 1 & 0 & 2 & -6 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -1 & -1 & 1 \\ -6 & 2 & 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 2 & 3 & 0 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 2 & 3 & 4 & -10 \end{bmatrix}$

6. $\begin{bmatrix} -1 \end{bmatrix}$

7. $\begin{bmatrix} 8 & 6 & 4 & 1 & 3 \\ 2 & 1 & -7 & 4 & 1 \\ 1 & 1 & -1 & 2 & 1 \\ 1 & -1 & 2 & 0 & 0 \end{bmatrix}$

8. $\begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$

In Exercises 9–14, determine whether the matrix is in row-echelon form. If it is, determine whether it is also in reduced row-echelon form.

9. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 1 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 0 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

In Exercises 15–22, find the solution set of the system of linear equations represented by the augmented matrix.

15. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

16. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

17. $\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

18. $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

19. $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

20. $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & -2 & 1 & -2 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

21. $\begin{bmatrix} 1 & 2 & 0 & 1 & 4 \\ 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

22. $\begin{bmatrix} 1 & 2 & 0 & 1 & 3 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

In Exercises 23–36, solve the system using either Gaussian elimination with back-substitution or Gauss-Jordan elimination.

23. $\begin{aligned} x + 2y &= 7 \\ 2x + y &= 8 \end{aligned}$

24. $\begin{aligned} 2x + 6y &= 16 \\ -2x - 6y &= -16 \end{aligned}$

25. $\begin{aligned} -x + 2y &= 1.5 \\ 2x - 4y &= 3 \end{aligned}$

26. $\begin{aligned} 2x - y &= -0.1 \\ 3x + 2y &= 1.6 \end{aligned}$

27. $\begin{aligned} -3x + 5y &= -22 \\ 3x + 4y &= 4 \\ 4x - 8y &= 32 \end{aligned}$

28. $\begin{aligned} x + 2y &= 0 \\ x + y &= 6 \\ 3x - 2y &= 8 \end{aligned}$

29. $\begin{aligned} x_1 \qquad - 3x_3 &= -2 \\ 3x_1 + x_2 - 2x_3 &= 5 \\ 2x_1 + 2x_2 + x_3 &= 4 \end{aligned}$

30. $\begin{aligned} 2x_1 - x_2 + 3x_3 &= 24 \\ 2x_2 - x_3 &= 14 \\ 7x_1 - 5x_2 \qquad &= 6 \end{aligned}$

31. $\begin{aligned} x_1 + x_2 - 5x_3 &= 3 \\ x_1 \qquad - 2x_3 &= 1 \\ 2x_1 - x_2 - x_3 &= 0 \end{aligned}$

32. $\begin{aligned} 2x_1 \qquad + 3x_3 &= 3 \\ 4x_1 - 3x_2 + 7x_3 &= 5 \\ 8x_1 - 9x_2 + 15x_3 &= 10 \end{aligned}$

33. $\begin{aligned} 4x + 12y - 7z - 20w &= 22 \\ 3x + 9y - 5z - 28w &= 30 \end{aligned}$

34. $\begin{aligned} x + 2y + z &= 8 \\ -3x - 6y - 3z &= -21 \end{aligned}$

35. $\begin{aligned} 3x + 3y + 12z &= 6 \\ x + y + 4z &= 2 \\ 2x + 5y + 20z &= 10 \\ -x + 2y + 8z &= 4 \end{aligned}$

36. $\begin{aligned} 2x + y - z + 2w &= -6 \\ 3x + 4y \qquad + w &= 1 \\ x + 5y + 2z + 6w &= -3 \\ 5x + 2y - z - w &= 3 \end{aligned}$

In Exercises 37–42, use a computer software program or graphing utility to solve the system of linear equations.

37. $\begin{aligned} x_1 - 2x_2 + 5x_3 - 3x_4 &= 23.6 \\ x_1 + 4x_2 - 7x_3 - 2x_4 &= 45.7 \\ 3x_1 - 5x_2 + 7x_3 + 4x_4 &= 29.9 \end{aligned}$

38. $\begin{aligned} 23.4x - 45.8y + 43.7z &= 87.2 \\ 86.4x + 12.3y - 56.9z &= 14.5 \\ 93.6x - 50.7y + 12.6z &= 44.4 \end{aligned}$

39.
$$\begin{aligned} x_1 - x_2 + 2x_3 + 2x_4 + 6x_5 &= 6 \\ 3x_1 - 2x_2 + 4x_3 + 4x_4 + 12x_5 &= 14 \\ x_2 - x_3 - x_4 - 3x_5 &= -3 \\ 2x_1 - 2x_2 + 4x_3 + 5x_4 + 15x_5 &= 10 \\ 2x_1 - 2x_2 + 4x_3 + 4x_4 + 13x_5 &= 13 \end{aligned}$$

40.
$$\begin{aligned} x_1 + x_2 - 2x_3 + 3x_4 + 2x_5 &= 9 \\ 3x_1 + 3x_2 - x_3 + x_4 + x_5 &= 5 \\ 2x_1 + 2x_2 - x_3 + x_4 - 2x_5 &= 1 \\ 4x_1 + 4x_2 + x_3 \qquad - 3x_5 &= 4 \\ 8x_1 + 5x_2 - 2x_3 - x_4 + 2x_5 &= 3 \end{aligned}$$

41.
$$\begin{aligned} 4x_1 - 3x_2 + x_3 - x_4 + 2x_5 - x_6 &= 8 \\ x_1 - 2x_2 + x_3 - 3x_4 + x_5 - 4x_6 &= 4 \\ 2x_1 + x_2 - 3x_3 + x_4 - 2x_5 + 5x_6 &= 2 \\ -2x_1 + 3x_2 - x_3 + x_4 - x_5 + 2x_6 &= -7 \\ x_1 - 3x_2 + x_3 - 2x_4 + x_5 - 2x_6 &= 9 \\ 5x_1 - 4x_2 - x_3 - x_4 + 4x_5 + 5x_6 &= 9 \end{aligned}$$

42.
$$\begin{aligned} x_1 + 2x_2 - 2x_3 + 2x_4 - x_5 + 3x_6 &= 0 \\ 2x_1 - x_2 + 3x_3 + x_4 - 3x_5 + 2x_6 &= 17 \\ x_1 + 3x_2 - 2x_3 + x_4 - 2x_5 - 3x_6 &= -5 \\ 3x_1 - 2x_2 + x_3 - x_4 + 3x_5 - 2x_6 &= -1 \\ -x_1 - 2x_2 + x_3 + 2x_4 - 2x_5 + 3x_6 &= 10 \\ x_1 - 3x_2 + x_3 + 3x_4 - 2x_5 + x_6 &= 11 \end{aligned}$$

In Exercises 43–46, solve the homogeneous linear system corresponding to the coefficient matrix provided.

43. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ **44.** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

45. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **46.** $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

47. Consider the matrix $A = \begin{bmatrix} 1 & k & 2 \\ -3 & 4 & 1 \end{bmatrix}$.

(a) If A is the *augmented* matrix of a system of linear equations, determine the number of equations and the number of variables.

(b) If A is the *augmented* matrix of a system of linear equations, find the value(s) of k such that the system is consistent.

(c) If A is the *coefficient* matrix of a *homogeneous* system of linear equations, determine the number of equations and the number of variables.

(d) If A is the *coefficient* matrix of a *homogeneous* system of linear equations, find the value(s) of k such that the system is consistent.

48. Consider the matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 2 & k \\ 4 & -2 & 6 \end{bmatrix}$.

(a) If A is the *augmented* matrix of a system of linear equations, determine the number of equations and the number of variables.

(b) If A is the *augmented* matrix of a system of linear equations, find the value(s) of k such that the system is consistent.

(c) If A is the *coefficient* matrix of a *homogeneous* system of linear equations, determine the number of equations and the number of variables.

(d) If A is the *coefficient* matrix of a *homogeneous* system of linear equations, find the value(s) of k such that the system is consistent.

In Exercises 49 and 50, find values of a, b, and c (if possible) such that the system of linear equations has (a) a unique solution, (b) no solution, and (c) an infinite number of solutions.

49.
$$\begin{aligned} x + y \qquad &= 2 \\ y + z &= 2 \\ x \qquad + z &= 2 \\ ax + by + cz &= 0 \end{aligned}$$

50.
$$\begin{aligned} x + y \qquad &= 0 \\ y + z &= 0 \\ x \qquad + z &= 0 \\ ax + by + cz &= 0 \end{aligned}$$

51. The system below has one solution: $x = 1$, $y = -1$, and $z = 2$.

$4x - 2y + 5z = 16$	**Equation 1**
$x + y = 0$	**Equation 2**
$-x - 3y + 2z = 6$	**Equation 3**

Solve the systems provided by (a) Equations 1 and 2, (b) Equations 1 and 3, and (c) Equations 2 and 3. (d) How many solutions does each of these systems have?

52. Assume the system below has a unique solution.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$	**Equation 1**
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$	**Equation 2**
$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$	**Equation 3**

Does the system composed of Equations 1 and 2 have a unique solution, no solution, or an infinite number of solutions?

In Exercises 53 and 54, find the unique reduced row-echelon matrix that is row-equivalent to the matrix provided.

53. $\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}$

54. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

55. Writing Describe all possible 2×2 reduced row-echelon matrices. Support your answer with examples.

56. Writing Describe all possible 3×3 reduced row-echelon matrices. Support your answer with examples.

True or False? In Exercises 57 and 58, determine whether each statement is true or false. If a statement is true, give a reason or cite an appropriate statement from the text. If a statement is false, provide an example that shows the statement is not true in all cases or cite an appropriate statement from the text.

57. (a) A 6×3 matrix has six rows.

(b) Every matrix is row-equivalent to a matrix in row-echelon form.

(c) If the row-echelon form of the augmented matrix of a system of linear equations contains the row $[1\ 0\ 0\ 0\ 0]$, then the original system is inconsistent.

(d) A homogeneous system of four linear equations in six variables has an infinite number of solutions.

58. (a) A 4×7 matrix has four columns.

(b) Every matrix has a unique reduced row-echelon form.

(c) A homogeneous system of four linear equations in four variables is always consistent.

(d) Multiplying a row of a matrix by a constant is one of the elementary row operations.

In Exercises 59 and 60, determine conditions on a, b, c, and d such that the matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

will be row-equivalent to the given matrix.

59. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

60. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

In Exercises 61 and 62, find all values of λ (the Greek letter lambda) such that the homogeneous system of linear equations will have nontrivial solutions.

61. $\begin{aligned} (\lambda - 2)x + \quad\quad y &= 0 \\ x + (\lambda - 2)y &= 0 \end{aligned}$

62. $\begin{aligned} (\lambda - 1)x + 2y &= 0 \\ x + \lambda y &= 0 \end{aligned}$

63. Writing Is it possible for a system of linear equations with fewer equations than variables to have no solution? If so, give an example.

64. Writing Does a matrix have a unique row-echelon form? Illustrate your answer with examples. Is the reduced row-echelon form unique?

65. Writing Consider the 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Perform the sequence of row operations.

(a) Add (-1) times the second row to the first row.

(b) Add 1 times the first row to the second row.

(c) Add (-1) times the second row to the first row.

(d) Multiply the first row by (-1).

What happened to the original matrix? Describe, in general, how to interchange two rows of a matrix using only the second and third elementary row operations.

66. The augmented matrix represents a system of linear equations that has been reduced using Gauss-Jordan elimination. Write a system of equations with nonzero coefficients that is represented by the reduced matrix.

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

There are many correct answers.

67. Writing Describe the row-echelon form of an augmente matrix that corresponds to a system of linear equations that inconsistent.

68. Writing Describe the row-echelon form of an augmente matrix that corresponds to a system of linear equations that h infinitely many solutions.

69. Writing In your own words, describe the difference betwe a matrix in row-echelon form and a matrix in reduc row-echelon form.

Using Gauss-Jordan elimination, a graphing utility, or a computer software program, you can solve this system to obtain

$$I_1 = 1, \quad I_2 = 2, \quad I_3 = 1, \quad I_4 = 1, \quad I_5 = 3, \quad \text{and} \quad I_6 = 2$$

meaning $I_1 = 1$ amp, $I_2 = 2$ amps, $I_3 = 1$ amp, $I_4 = 1$ amp, $I_5 = 3$ amps, and $I_6 = 2$ amps.

SECTION 1.3 Exercises

Polynomial Curve Fitting

In Exercises 1–6, (a) determine the polynomial function whose graph passes through the given points, and (b) sketch the graph of the polynomial function, showing the given points.

1. $(2, 5), (3, 2), (4, 5)$
2. $(2, 4), (3, 4), (4, 4)$
3. $(2, 4), (3, 6), (5, 10)$
4. $(-1, 3), (0, 0), (1, 1), (4, 58)$
5. $(2006, 5), (2007, 7), (2008, 12)$ $(z = x - 2007)$
6. $(2005, 150), (2006, 180), (2007, 240), (2008, 360)$ $(z = x - 2005)$

7. **Writing** Try to fit the graph of a polynomial function to the values shown in the table. What happens, and why?

x	1	2	3	3	4
y	1	1	2	3	4

8. The graph of a function f passes through the points $(0, 1)$, $(2, \frac{1}{3})$, and $(4, \frac{1}{5})$. Find a quadratic function whose graph passes through these points.

9. Find a polynomial function p of degree 2 or less that passes through the points $(0, 1)$, $(2, 3)$, and $(4, 5)$. Then sketch the graph of $y = 1/p(x)$ and compare this graph with the graph of the polynomial function found in Exercise 8.

10. **Calculus** The graph of a parabola passes through the points $(0, 1)$ and $(\frac{1}{2}, \frac{1}{2})$ and has a horizontal tangent at $(\frac{1}{2}, \frac{1}{2})$. Find an equation for the parabola and sketch its graph.

11. **Calculus** The graph of a cubic polynomial function has horizontal tangents at $(1, -2)$ and $(-1, 2)$. Find an equation for the cubic and sketch its graph.

12. Find an equation of the circle passing through the points $(1, 3)$, $(-2, 6)$, and $(4, 2)$.

13. The U.S. census lists the population of the United States as 227 million in 1980, 249 million in 1990, and 281 million in 2000. Fit a second-degree polynomial passing through these three points and use it to predict the population in 2010 and in 2020. (Source: U.S. Census Bureau)

14. The U.S. population figures for the years 1920, 1930, 1940, and 1950 are shown in the table. (Source: U.S. Census Bureau)

Year	1920	1930	1940	1950
Population (in millions)	106	123	132	151

(a) Find a cubic polynomial that fits these data and use it to estimate the population in 1960.

(b) The actual population in 1960 was 179 million. How does your estimate compare?

15. The net profits (in millions of dollars) for Microsoft from 2000 to 2007 are shown in the table. (Source: Microsoft Corporation)

Year	2000	2001	2002	2003
Net Profit	9421	10,003	10,384	10,526

Year	2004	2005	2006	2007
Net Profit	11,330	12,715	12,599	14,41

(a) Set up a system of equations to fit the data for the years 2001, 2003, 2005, and 2007 to a cubic model.

(b) Solve the system. Does the solution produce a reasonable model for predicting future net profits? Explain.

16. The sales (in billions of dollars) for Wal-Mart stores from 2000 to 2007 are shown in the table. (Source: Wal-Mart)

Year	2000	2001	2002	2003
Sales	191.3	217.8	244.5	256.3

Year	2004	2005	2006	2007
Sales	285.2	312.4	346.5	377.0

(a) Set up a system of equations to fit the data for the years 2001, 2003, 2005, and 2007 to a cubic model.
(b) Solve the system. Does the solution produce a reasonable model for predicting future sales? Explain.

17. Use $\sin 0 = 0$, $\sin(\pi/2) = 1$, and $\sin \pi = 0$ to estimate $\sin(\pi/3)$.

18. Use $\log_2 1 = 0$, $\log_2 2 = 1$, and $\log_2 4 = 2$ to estimate $\log_2 3$.

19. Guided Proof Prove that if a polynomial function $p(x) = a_0 + a_1x + a_2x^2$ is zero for $x = -1$, $x = 0$, and $x = 1$, then $a_0 = a_1 = a_2 = 0$.

Getting Started: Write a system of linear equations and solve the system for a_0, a_1, and a_2.

(i) Substitute $x = -1, 0$, and 1 into $p(x)$.
(ii) Set the result equal to 0.
(iii) Solve the resulting system of linear equations in the variables a_0, a_1, and a_2.

20. The statement in Exercise 19 can be generalized: If a polynomial function $p(x) = a_0 + a_1x + \cdots + a_{n-1}x^{n-1}$ is zero for more than $n - 1$ x-values, then $a_0 = a_1 = \cdots = a_{n-1} = 0$. Use this result to prove that there is at most one polynomial function of degree $n - 1$ (or less) whose graph passes through n points in the plane with distinct x-coordinates.

Network Analysis

21. Water is flowing through a network of pipes (in thousands of cubic meters per hour), as shown in Figure 1.15.

(a) Solve this system for the water flow represented by x_i, $i = 1, 2, \ldots, 7$.
(b) Find the water flow when $x_6 = x_7 = 0$.
(c) Find the water flow when $x_5 = 1000$ and $x_6 = 0$.

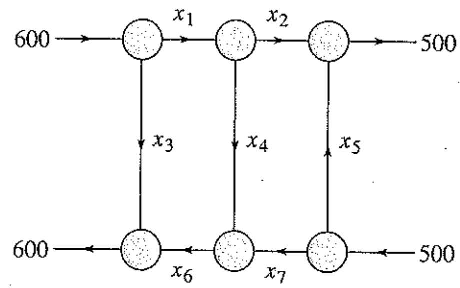

Figure 1.15

22. The flow of traffic (in vehicles per hour) through a network of streets is shown in Figure 1.16.

(a) Solve this system for x_i, $i = 1, 2, \ldots, 5$.
(b) Find the traffic flow when $x_2 = 200$ and $x_3 = 50$.
(c) Find the traffic flow when $x_2 = 150$ and $x_3 = 0$.

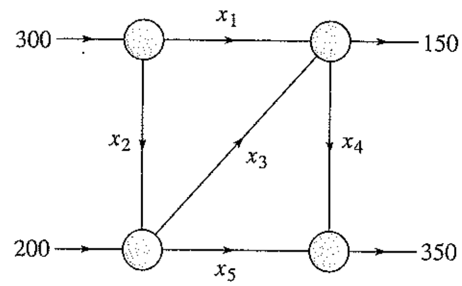

Figure 1.16

23. The flow of traffic (in vehicles per hour) through a network of streets is shown in Figure 1.17.

(a) Solve this system for x_i, $i = 1, 2, 3, 4$.
(b) Find the traffic flow when $x_4 = 0$.
(c) Find the traffic flow when $x_4 = 100$.

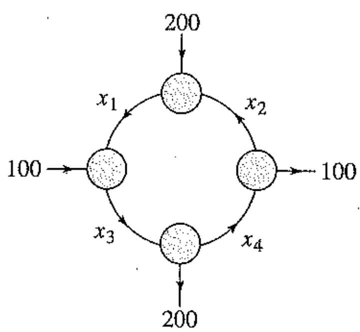

Figure 1.17

24. The flow of traffic (in vehicles per hour) through a network of streets is shown in Figure 1.18.

(a) Solve this system for x_i, $i = 1, 2, \ldots, 5$.
(b) Find the traffic flow when $x_3 = 0$ and $x_5 = 100$.
(c) Find the traffic flow when $x_3 = x_5 = 100$.

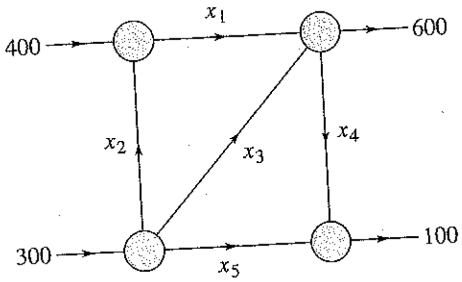

Figure 1.18

25. Determine the currents I_1, I_2, and I_3 for the electrical network shown in Figure 1.19.

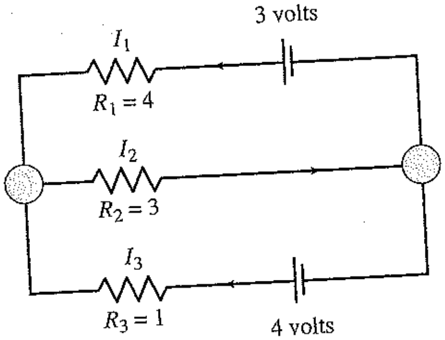

Figure 1.19

26. Determine the currents I_1, I_2, and I_3 for the electrical network shown in Figure 1.20.

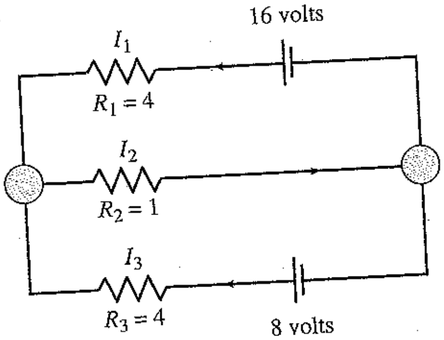

Figure 1.20

27. (a) Determine the currents I_1, I_2, and I_3 for the electrical network shown in Figure 1.21.

(b) How is the result affected when A is changed to 2 volts and B is changed to 6 volts?

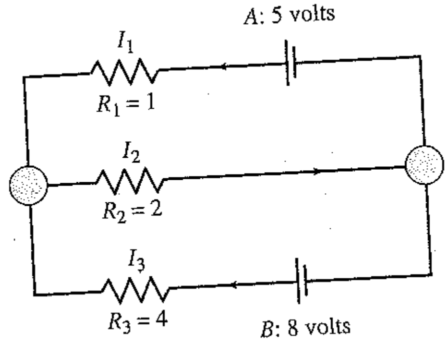

Figure 1.21

28. Determine the currents I_1, I_2, I_3, I_4, I_5, and I_6 for the electrica network shown in Figure 1.22.

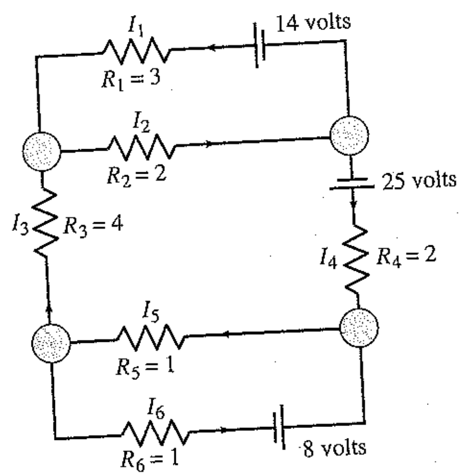

Figure 1.22

In Exercises 29–32, use a system of equations to write the pa fraction decomposition of the rational expression. Then solve system using matrices.

29. $\dfrac{4x^2}{(x+1)^2(x-1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

30. $\dfrac{8x^2}{(x-1)^2(x+1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

31. $\dfrac{20 - x^2}{(x+2)(x-2)^2} = \dfrac{A}{x+2} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$

32. $\dfrac{3x^2 - 7x - 12}{(x+4)(x-4)^2} = \dfrac{A}{x+4} + \dfrac{B}{x-4} + \dfrac{C}{(x-4)^2}$

In Exercises 33 and 34, find the values of x, y, and λ that satisfy the system of equations. Such systems arise in certain problems of calculus, and λ is called the Lagrange multiplier.

33. $\begin{aligned} 2x \quad\quad + \lambda &= 0 \\ 2y + \lambda &= 0 \\ x + y \quad\quad - 4 &= 0 \end{aligned}$

34. $\begin{aligned} 2y + 2\lambda + 2 &= 0 \\ 2x \quad\quad + \lambda + 1 &= 0 \\ 2x + y \quad\quad - 100 &= 0 \end{aligned}$

35. In Super Bowl XLI on February 4, 2007, the Indianapolis Colts beat the Chicago Bears by a score of 29 to 17. The total points scored came from 13 scoring plays, which were a combination of touchdowns, extra-point kicks, and field goals, worth 6, 1, and 3 points, respectively. The numbers of field goals and extra-point kicks were equal. Write a system of equations to represent this event. Then determine the number of each type of scoring play. (Source: National Football League)

36. In the 2007 Fiesta Bowl Championship Series on January 8, 2007, the University of Florida Gators defeated the Ohio State University Buckeyes by a score of 41 to 14. The total points scored came from a combination of touchdowns, extra-point kicks, and field goals, worth 6, 1, and 3 points, respectively. The numbers of touchdowns and extra-point kicks were equal. The number of touchdowns was one more than three times the number of field goals. Write a system of equations to represent this event. Then determine the number of each type of scoring play. (Source: www.fiestabowl.org)

CHAPTER 1 Review Exercises

In Exercises 1–8, determine whether the equation is linear in the variables x and y.

1. $2x - y^2 = 4$
2. $2xy - 6y = 0$
3. $(\sin\pi)x + y = 2$
4. $e^{-2}x + 5y = 8$
5. $\dfrac{2}{x} + 4y = 3$
6. $\dfrac{4}{y} - x = 10$
7. $\frac{1}{2}x - \frac{1}{4}y = 0$
8. $\frac{3}{5}x + \frac{7}{10}y = 2$

In Exercises 9 and 10, find a parametric representation of the solution set of the linear equation.

9. $-4x + 2y - 6z = 1$
10. $3x_1 + 2x_2 - 4x_3 = 0$

In Exercises 11–22, solve the system of linear equations.

11. $\begin{aligned} x + y &= 2 \\ 3x - y &= 0 \end{aligned}$

12. $\begin{aligned} x + y &= -1 \\ 3x + 2y &= 0 \end{aligned}$

13. $\begin{aligned} 3y &= 2x \\ y &= x + 4 \end{aligned}$

14. $\begin{aligned} x &= y + 3 \\ 4x &= y + 10 \end{aligned}$

15. $\begin{aligned} y + x &= 0 \\ 2x + y &= 0 \end{aligned}$

16. $\begin{aligned} y &= -4x \\ y &= x \end{aligned}$

17. $\begin{aligned} x - y &= 9 \\ -x + y &= 1 \end{aligned}$

18. $\begin{aligned} 40x_1 + 30x_2 &= 24 \\ 20x_1 + 15x_2 &= -14 \end{aligned}$

19. $\begin{aligned} 0.2x_1 + 0.3x_2 &= 0.14 \\ 0.4x_1 + 0.5x_2 &= 0.20 \end{aligned}$

20. $\begin{aligned} 0.2x - 0.1y &= 0.07 \\ 0.4x - 0.5y &= -0.01 \end{aligned}$

21. $\begin{aligned} \tfrac{1}{2}x - \tfrac{1}{3}y &= 0 \\ 3x + 2(y + 5) &= 10 \end{aligned}$

22. $\begin{aligned} \tfrac{1}{3}x + \tfrac{4}{7}y &= 3 \\ 2x + 3y &= 15 \end{aligned}$

In Exercises 23 and 24, determine the size of the matrix.

23. $\begin{bmatrix} 2 & 3 & -1 \\ 0 & 5 & 1 \end{bmatrix}$

24. $\begin{bmatrix} 2 & 1 \\ -4 & -1 \\ 0 & 5 \end{bmatrix}$

In Exercises 25–28, determine whether the matrix is in row-echelon form. If it is, determine whether it is also in reduced row-echelon form.

25. $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

26. $\begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

27. $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

28. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

In Exercises 29 and 30, find the solution set of the system of linear equations represented by the augmented matrix.

29. $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

30. $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

In Exercises 31–40, solve the system using either Gaussian elimination with back-substitution or Gauss-Jordan elimination.

31. $\begin{aligned} -x + y + 2z &= 1 \\ 2x + 3y + z &= -2 \\ 5x + 4y + 2z &= 4 \end{aligned}$

32. $\begin{aligned} 2x + 3y + z &= 10 \\ 2x - 3y - 3z &= 22 \\ 4x - 2y + 3z &= -2 \end{aligned}$

33. $\begin{aligned} 2x + 3y + 3z &= 3 \\ 6x + 6y + 12z &= 13 \\ 12x + 9y - z &= 2 \end{aligned}$

34. $\begin{aligned} 2x \quad\quad + 6z &= -9 \\ 3x - 2y + 11z &= -16 \\ 3x - y + 7z &= -11 \end{aligned}$