Lecture

Introduction. Motivation for the Course.

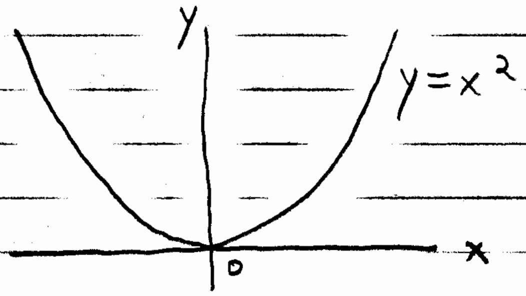
$f(x) = x^2$ is an example of

A REAL-VALUED FUNCTION OF A SINGLE VARIABLE

Plug in a value of x and get back a value of the function, e.g.,

$$x = 2 \Rightarrow f(2) = 4$$

- $f(x) = x^2$ has a *graph* (a CURVE):



(Here y is set equal to $f(x)$.)

- $f(x) = x^2$ has a *derivative*:

$$\frac{d}{dx}(x^2) = 2x$$

- $f(x) = x^2$ also has an <u>integral</u>:

$$\int x^2 dx = \frac{x^3}{3} + C.$$

In this course, you are going to learn how to work with

1. REAL-VALUED FUNCTIONS OF MORE THAN ONE VARIABLE, e.g., $f(x,y) = x^2 + y^2$

2. VECTOR-VALUED FUNCTIONS OF MORE THAN ONE VARIABLE, e.g., $f(x,y) = (x^2, y^2)$

With $f(x,y) = x^2 + y^2$, you can plug in a value of x <u>and</u> of y and get back a value of the function, e.g.,

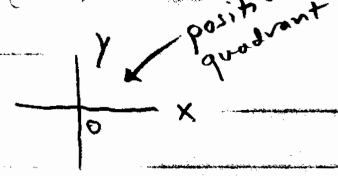
$$(x,y) = (1,2) \Rightarrow f(1,2) = 1^2 + 2^2 = 5$$

- $f(x,y) = x^2 + y^2$ has a <u>graph</u> (a SURFACE):

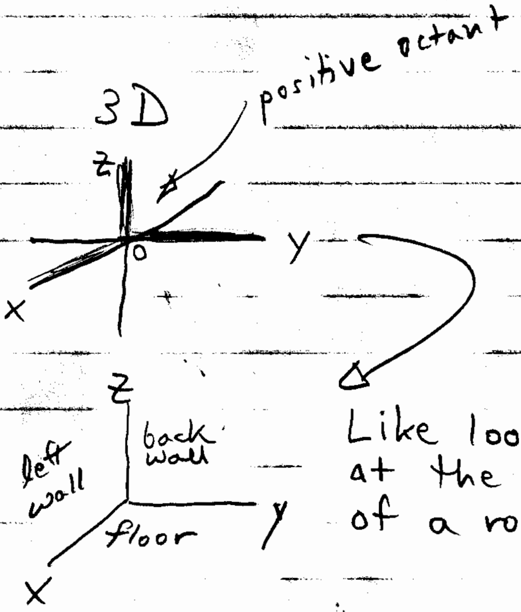
① Primitive Method for Graphing

- by hand
- use prior knowledge of 2D graphing. (for functions of the form $y = f(x)$) and graph 3D surface in horizontal or vertical slices

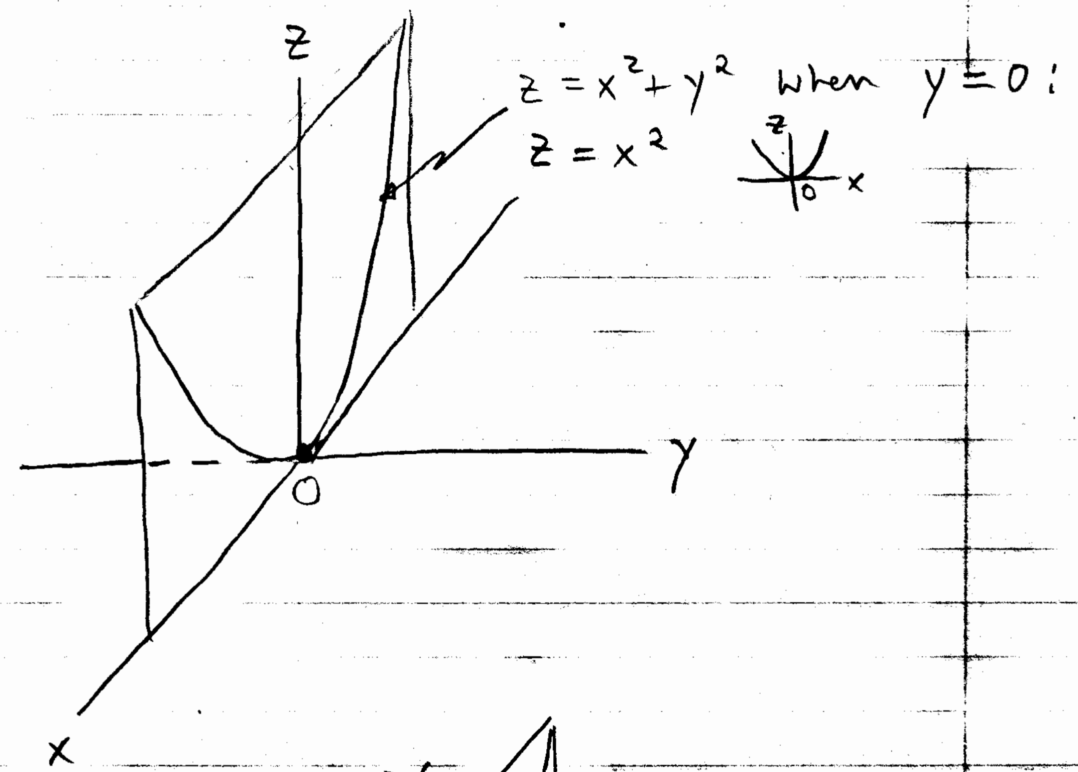
2D

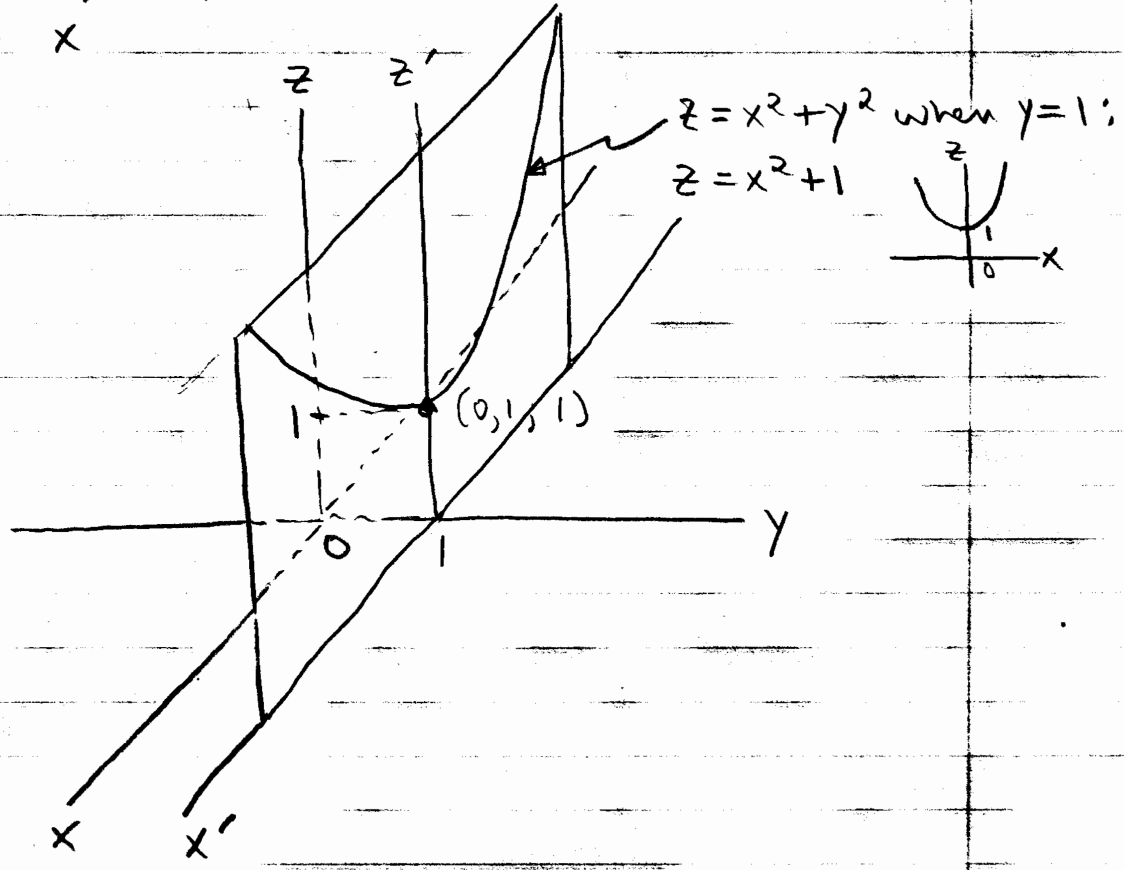


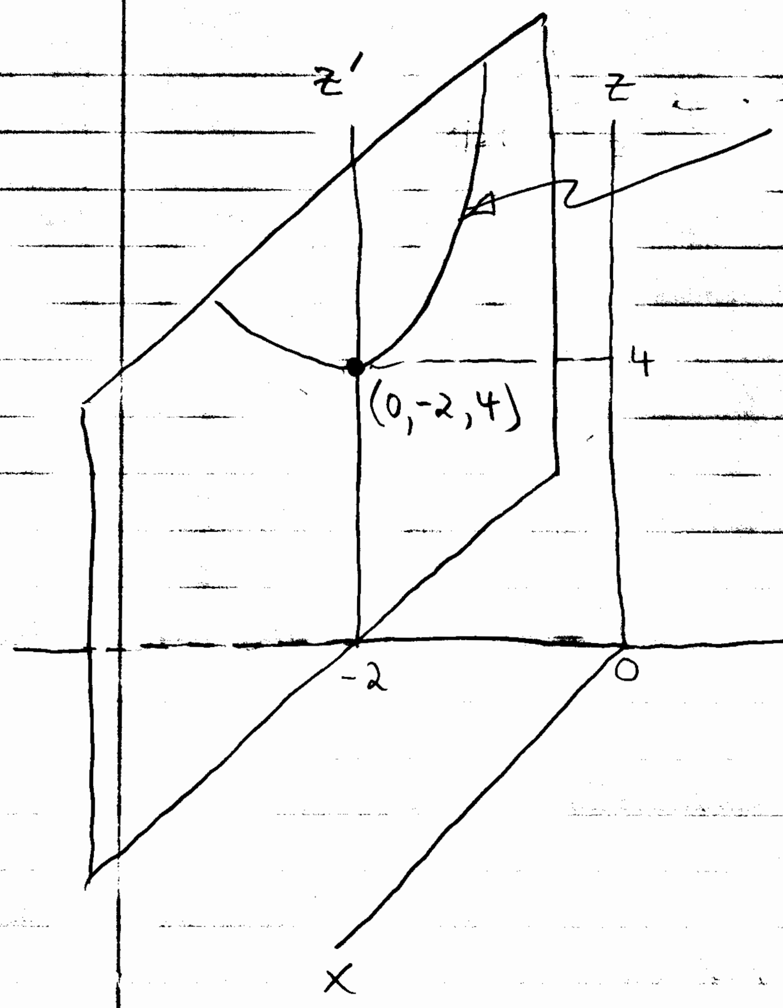
3D



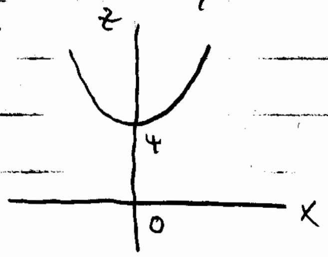
Like looking at the corner of a room


z
z = x² + y² when y = 0:
z = x²
z
0
x
0
y
x

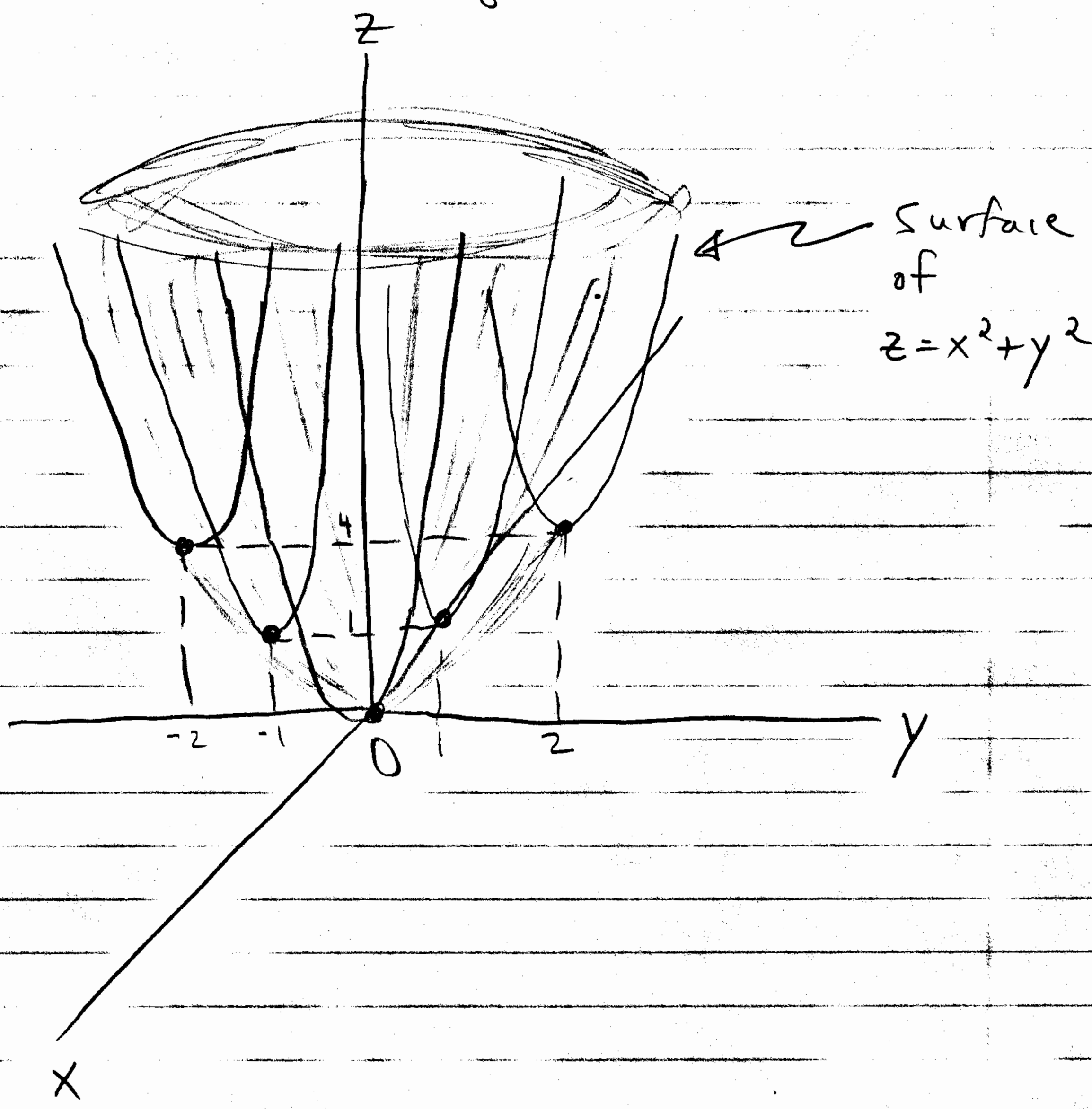

z
z'
z = x² + y² when y = 1:
z = x² + 1
z
1
0
x
1
(0, 1, 1)
0
1
y
x
x'



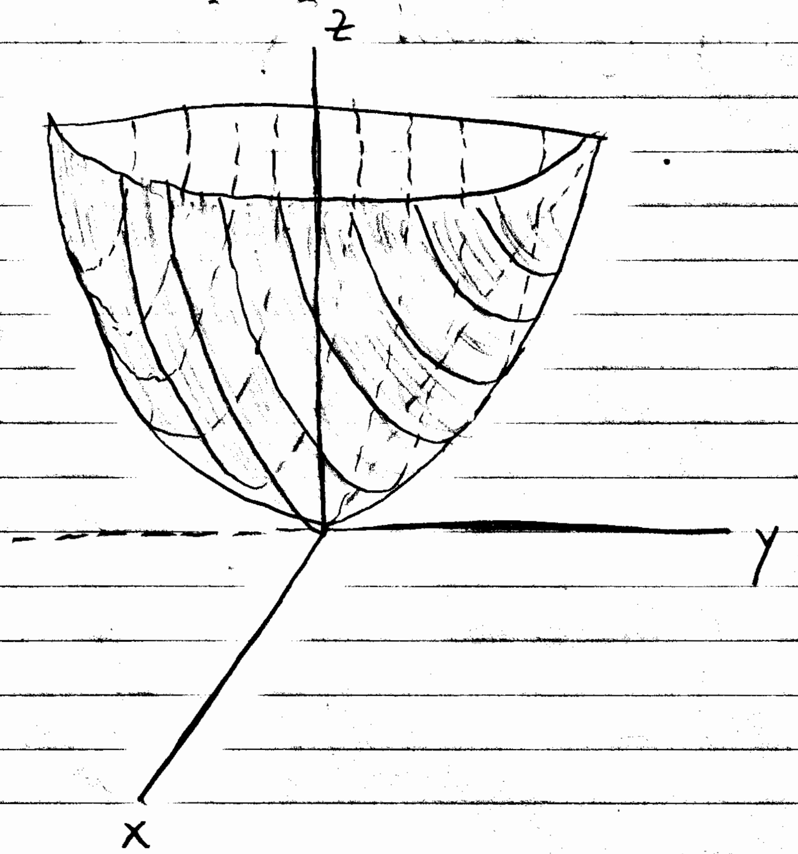
$z = x^2 + y^2$ when $y = -2$:
$z = x^2 + 4$



Put all of these slices together to get something like



<u>or</u>


z
y
x

② "Low-Tech" Method for Graphing – use TI-89 3D graphics function

[ON] → [MODE] → [D] → [5] → [ENTER]

3D (under [5])

→ [◇] → [F1] → [X] → [^] → [2]

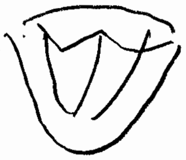
(GREEN BUTTON) (under [◇]); $Z1 =$ (under [F1]); $X \wedge 2$ (under [2])

→ [+] → [Y] → [^] → [2] → [ENTER]

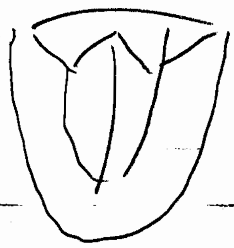
$Y \wedge 2$ (under [2]); $z = x^2 + y^2$

→ [F2] → [6] → [×] → [×]

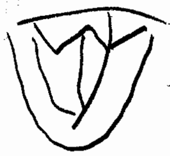
(TIMES BUTTON) (under the first [×])



TI-89 actually draws vertical slices and puts these together to form 3D sketch



Expanded sketch



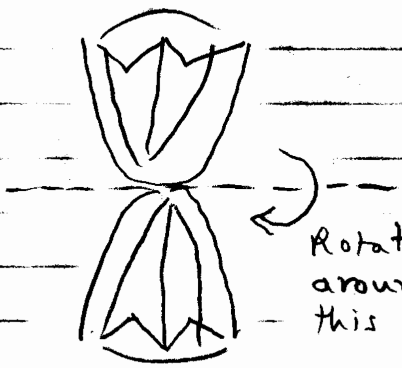
Back-to-normal sketch

→ ▽ → ENTER → HOME

HOLD DOWN FOR ~2 SEC TO ANIMATE SKETCH

STOPS ANIMATION (Press ENTER again and animation will start up again)

BRINGS YOU BACK TO HOME SCREEN



Rotates around this axis

③ "High-Tech" Method for Graphing

- use DPGraph, which is software that produces 3D graphs which can be animated

Available in one of two ways.

1. Through Prof. Hassan Sedaghat's website ;

May not work however

http://www.people.vcu.edu/~hsedagha

↓

Spring 2003 courses :

Math 307: Multivariable Calculus ← CLICK ON

↓

Software ← CLICK ON

↓

VCU Math 307: Software related.

⋮

Required Software : DPGraph

⋮

Access. ... by clicking here and ...

CLICK ON

2. Through DPGraph's website:

http://www.dpgraph.com/subscribers.html

↓

Subscribing institutions sorted by name: ... V ...

CLICK ONCE ON this for "VCU"

↓

V

Virginia Commonwealth University, 23284 US

CLICK ONCE ON

↓

Save to Disk (C: Drive)

↓

SAVE AS
Save in: Desktop
filename: InstallDPGraph.exe
from www.davidparker.com

C:\WINDOWS\Desktop

↓

Licensed to install DPGraph?
Yes ...

CLICK ONCE ON

↓

OK to install now?
OK ...

CLICK ONCE ON

↓

Done.

↓

CLICK ON ALL { START → PROGRAMS → DPGRAPH → DPGRAPH

↓

Edit

CLICK ON

↓

⋮
graph3d(x^2 + y^2 = z)

ERASE WHAT IS PRESENT AND TYPE NEW EQUATION (with "= z") IN

Execute

↓

Background

CLICK ON

↓

⋮
⊙ White
⋮

BULLET

↓

Clipboard

CLICK ON

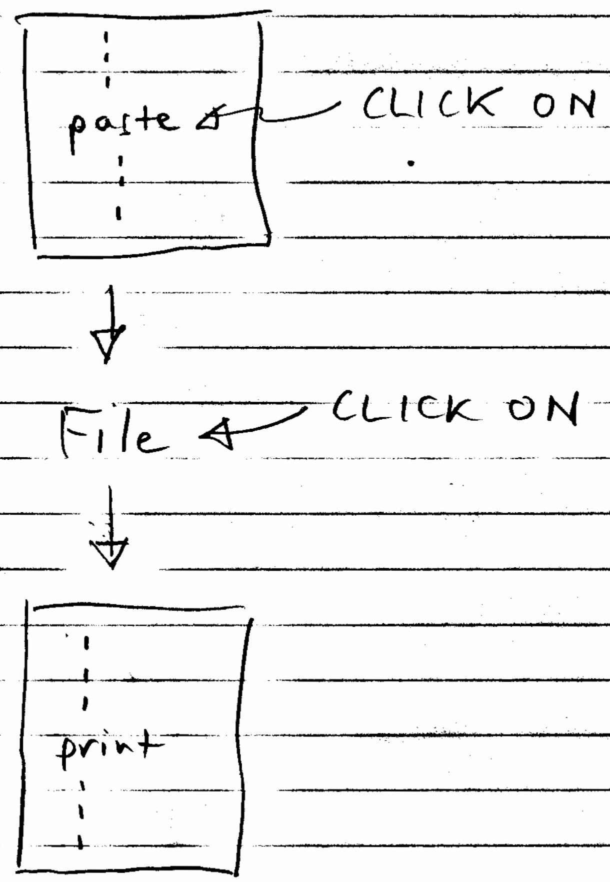
OK ...   CLICK ON

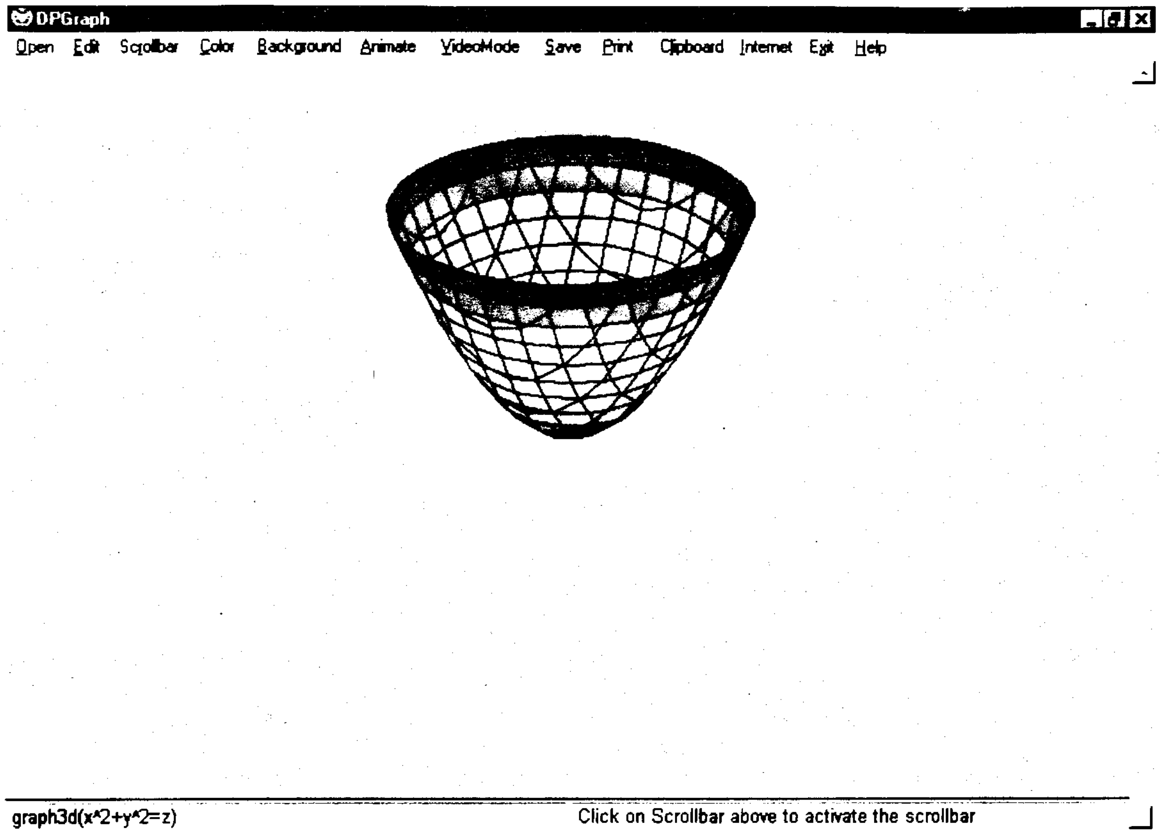
Alt   Print Screen / SysRq

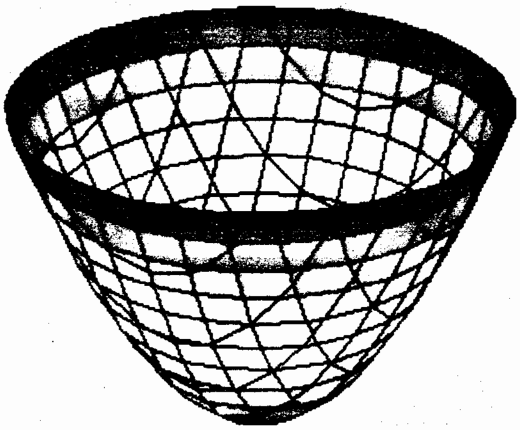
PRESS SIMULTANEOUSLY

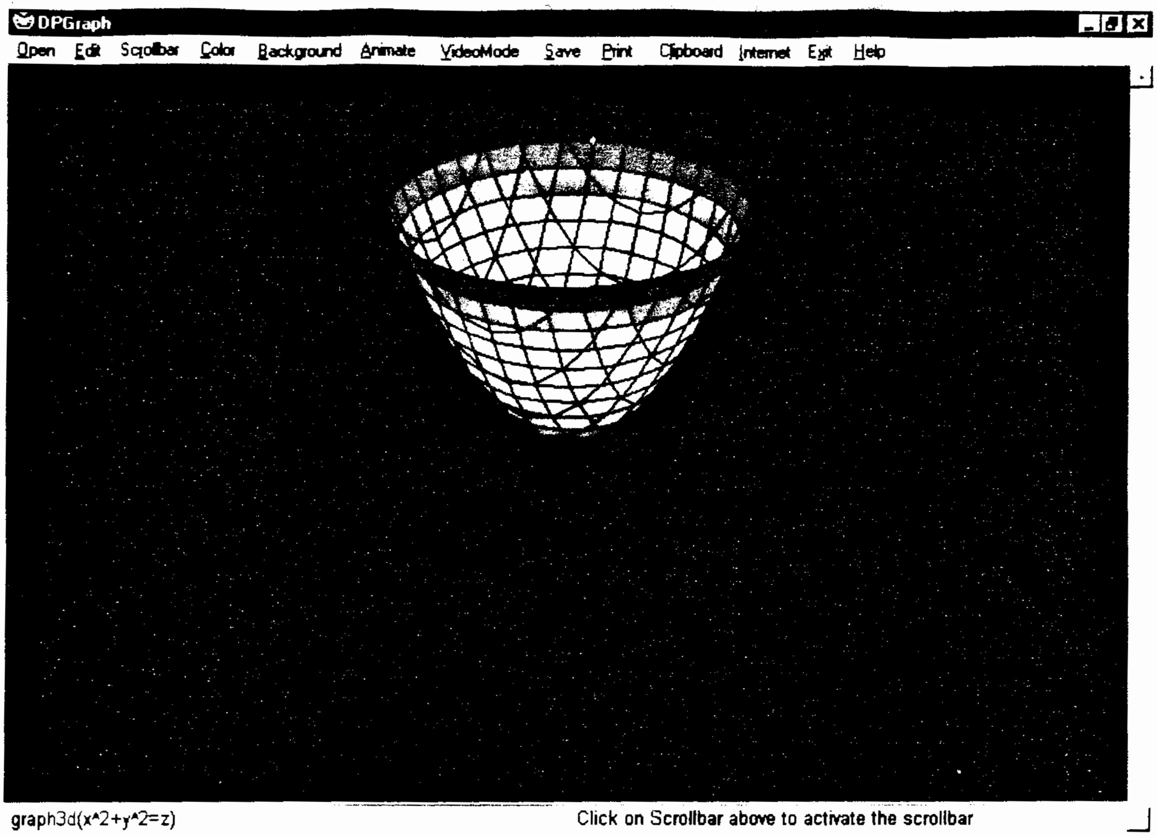
CLICK ON ALL { START → PROGRAMS → MICROSOFT WORD

Edit   CLICK ON


paste
CLICK ON
File
CLICK ON
print


DPGraph
Open Edit Scrollbar Color Background Animate VideoMode Save Print Clipboard Internet Exit Help
graph3d(x^2+y^2=z)
Click on Scrollbar above to activate the scrollbar




DPGraph
Open Edit Scrollbar Color Background Animate VideoMode Save Print Clipboard Internet Exit Help
graph3d(x^2+y^2=z)
Click on Scrollbar above to activate the scrollbar

- $f(x, y) = x^2 + y^2$ has <u>partial derivatives</u>:

Recall: $f(x) = x^2$

$$\frac{d}{dx}(x^2) = 2x$$

Here: $f(x, y) = x^2 + y^2$

$$\frac{\partial}{\partial x}(x^2+y^2) = \underbrace{\frac{\partial}{\partial x}(x^2)}_{\text{like } \frac{d}{dx}(x^2)} + \underbrace{\frac{\partial}{\partial x}(y^2)}_{\text{like } \frac{d}{dx}(\text{constant})}$$

The "partial of f with respect to x"

$$= 2x + 0$$

$$= 2x$$

$$\frac{\partial}{\partial y}(x^2+y^2) = \underbrace{\frac{\partial}{\partial y}(x^2)}_{\text{like } \frac{d}{dy}(\text{constant})} + \underbrace{\frac{\partial}{\partial x}(y^2)}_{\text{like } \frac{d}{dy}(y^2)}$$

The "partial of f with respect to y"

$$= 0 + 2y$$

$$= 2y$$

Also: $f(x,y) = x^2 + y^2$ has a "total derivative" (or "total differential") that puts together these partial derivatives

SEE Section 11.4
(which we are going to SKIP but which I am willing to go over with anyone who is interested)

- $f(x,y) = x^2 + y^2$ has "partial integrals":

SEE MATH 301 (Differential Equations) — Exact Equations

Also: $f(x,y) = x^2 + y^2$ has a "total integral"

SEE Chapter 12 — Double Integrals

ASIDE and OPTIONAL !!! :

In an ADVANCED CALCULUS course (which covers a lot of the theory behind CALCULUS I, II, III) and in Section 12.9 of our text to some extent (which we will not cover), the "derivative" of a real-valued or vector-valued function of two or more variables is defined.

Let

$$f(x_1, x_2, \ldots, x_n) = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \vdots \\ f_m(x_1, x_2, \ldots, x_n) \end{pmatrix}$$

be a vector-valued function of n variables. E.g.,

$$f(x_1, x_2) = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \\ f_3(x_1, x_2) \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 \\ x_1^2 + e^{x_2} \\ \sin x_1 + \cos x_2 \end{pmatrix}.$$

Then the (TOTAL) DERIVATIVE OF f

or

DIFFERENTIAL OF f

or

JACOBIAN MATRIX OF f

is given by an $m \times n$ (m rows, n columns) matrix, whose elements are partial derivatives of $f_1, f_2, \ldots, f_m$ and which is denoted by Df:

$$Df = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}$$

E.g., with $f(x_1, x_2) = \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \\ f_3(x_1, x_2) \end{pmatrix}$

$$= \begin{pmatrix} 2x_1 + x_2 \\ x_1^2 + e^{x_2} \\ \sin x_1 + \cos x_2 \end{pmatrix},$$

$$Df = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 2x_1 & e^{x_2} \\ \cos x_1 & -\sin x_2 \end{pmatrix}$$

If the (TOTAL) DERIVATIVE is evaluated at a particular point $\vec{x}_0 = (a_1, a_2, \ldots, a_n)$, then it is denoted by

$$Df(\vec{x}_0) = Df(a_1, a_2, \ldots, a_n).$$

E.g., Evaluate Df at $\vec{x}_0 = (3, -5)$ when

$$f(x_1, x_2) = \begin{pmatrix} 2x_1 + x_2 \\ x_1^2 + e^{x_2} \\ \sin x_1 + \cos x_2 \end{pmatrix} :$$

$$Df(\vec{x_0}) = Df(\underset{x_1}{3}, \underset{x_2}{-5}) = \begin{pmatrix} 2 & 1 \\ \overset{3}{2x_1} & e^{\overset{-5}{x_2}} \\ \underset{3}{\cos x_1} & \underset{-5}{-\sin x_2} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 6 & e^{-5} \\ \cos(3) & -\sin(-5) \end{pmatrix}$$