Lecture

Section 7.5. Applications.

We can pull the ideas from Sections 7.1, 7.2, and 7.4 together and solve an initial value problem.

Strategy: Transform an IVP into an ALGEBRAIC PROBLEM.

$$\begin{tikzcd}
\text{DE in } t \arrow[r, "L"] & \text{ALGEBRAIC EQ. in } s \arrow[d, "\text{solve}"] \\
\text{solution in } t & \text{solution in } s \arrow[l, "L^{-1}"']
\end{tikzcd}$$

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<u>Example</u>. (HW Exercise 1, p. 301.)

Solve the IVP

$$\begin{cases} y' - y = 1, \\ y(0) = 0 \end{cases}$$

Let $L\{y(t)\} = Y(s)$

Then $L\{y'(t)\} = sY(s) - \underbrace{y(0)}_{=0}$ and $L^{-1}\{Y(s)\} = y(t)$.

<u>STEP 1</u>. "Laplace transform" $y' - y = 1$:

$$L\{y' - y\} = L\{1\} \Longrightarrow$$

$$L\{y'(t)\} - L\{y(t)\} = L\{1\} \Longrightarrow$$

By LINEARITY

$L\{1\} = \frac{1}{s}$ from TABLE, item #1

$$[sY(s) - \underbrace{y(0)}_{=0}] - Y(s) = \frac{1}{s} \Longrightarrow$$

$$sY(s) - Y(s) = \frac{1}{s} \Longrightarrow$$

$$Y(s)\,(s-1) = \frac{1}{s} \Longrightarrow$$

$$Y(s) = \underbrace{\frac{1}{s(s-1)}}$$

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1} \Longrightarrow$$

$$1 = A(s-1) + Bs$$

$s=0$: $\quad 1 = A(0-1) + B(0) \Longrightarrow$

$1 = -A \Longrightarrow$

$A = -1$

$s=1$: $\quad 1 = A(1-1) + B(1) \Longrightarrow$

$B = 1$

$$\therefore\ Y(s) = \frac{-1}{s} + \frac{1}{s-1}$$

<u>STEP 2</u>. "Inverse transform" $Y(s) = \frac{-1}{s} + \frac{1}{s-1}$:

$$L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{-1}{s} + \frac{1}{s-1}\right\} \Longrightarrow$$

— By LINEARITY

$$y(t) = -L^{-1}\left\{\frac{1}{s}\right\} + L^{-1}\left\{\frac{1}{s-1}\right\}$$

$L^{-1}\left\{\frac{1}{s}\right\} = 1$ and $L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$, $a = 1$, from TABLE, items # 1, 11

$$= -(1) + e^{1 \cdot t}$$

$$= \boxed{-1 + e^{t}}$$