

Lecture

Subsection 4.1.3. Nonhomogeneous Equations.

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A nonhomogeneous linear n th-order DE is of the form

$$a_n(x)y^{(n)} + \dots + a_1(x)y' + a_0(x)y = g(x) \text{ on } I$$

$g(x) \neq 0$ on I . (i.e., $g(x)$ is not identically equal to zero on I) \Rightarrow $g(x)$ is not equal to zero for all x in I \Rightarrow there is at least one x in I such that $g(x) \neq 0$.

Its general solution (i.e., the solution giving all-possible solutions of the DE) is of the form

$$y(x) = \underbrace{y_c(x)}_{\text{Complementary soln.}} + \underbrace{y_p(x)}_{\text{particular soln.}}$$

= general soln. of

the corresponding

homogeneous eq.
 $a_n(x)y^{(n)} + \dots + a_0(x)y = 0$

= $c_1 y_1(x) + \dots + c_n y_n(x)$

- any particular soln.

(i.e., without an arbitrary const. multiplying it) of

the nonhomogeneous eq.

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[PROOF omitted - SEE text for proof
using "differential operators."]

Examples.

1. (Exercise 38(a), p. 109.)

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By inspection [i.e., by making an educated guess], determine a particular solution of

$$y'' + 2y = 10$$

SUGGESTION: y_p will "look like" the function $g(x) = 10$.

In other words, y_p will be some constant function.

So, let $y_p(x) = A$.

Let $y_p(x) = A$. Substitute y_p and its derivatives into the DE $y'' + 2y = 10$ to find A :

$$\left. \begin{array}{l} y_p(x) = A \\ y_p'(x) = 0 \\ y_p''(x) = 0 \end{array} \right\} \rightarrow$$

$$y_p'' + 2y_p = 10 \Rightarrow$$

$$0 + 2A = 10 \Rightarrow$$

$$A = 5 \Rightarrow \boxed{y_p(x) = 5}$$

2. (Exercise 38(d), p. 109.)

By inspection, determine a particular solution of

$$y'' + 2y = 8x + 5.$$

SUGGESTION: y_p will "look like" the function $g(x) = 8x + 5$. In other words, y_p will be some linear function. So, let $y_p(x) = Ax + B$.

Let $y_p(x) = Ax + B$. Substitute y_p and its derivatives into the DE $y'' + 2y = 8x + 5$ to find A and B :

$$\left. \begin{array}{l} y_p(x) = Ax + B \\ y_p'(x) = A \\ y_p''(x) = 0 \end{array} \right\} \Rightarrow$$

$$-y_p'' + 2y_p = 8x + 5 \Rightarrow$$

$$0 + 2(Ax + B) = 8x + 5 \Rightarrow$$

$$2A + 2B = 8x + 5 \Rightarrow$$

$$\begin{array}{l} 2A = 8 \\ 2B = 5 \end{array} \quad \Rightarrow \quad \left. \begin{array}{l} A = 4 \\ B = \frac{5}{2} \end{array} \right\}$$

$$\begin{array}{l} A = 4 \\ B = \frac{5}{2} \end{array} \quad \Rightarrow \quad \left. \begin{array}{l} A \\ B \end{array} \right\}$$

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$$y_p(x) = 4x + \frac{5}{2}$$

3. (HW Exercise 33, p. 109.)

Verify that

$$y = c_1 e^{2x} + c_2 e^{5x} + 6e^x$$

is the general solution of

$$y'' - 7y' + 10y = 24e^x \text{ on } (-\infty, \infty)$$

$$y = \underbrace{c_1 e^{2x} + c_2 e^{5x}}_{= y_c} + \underbrace{6e^x}_{= y_p}$$

① Show y_c is a soln. of $y'' - 7y' + 10y = 0$:

$$\left. \begin{aligned} y_c &= c_1 e^{2x} + c_2 e^{5x} \\ y'_c &= 2c_1 e^{2x} + 5c_2 e^{5x} \\ y''_c &= 4c_1 e^{2x} + 25c_2 e^{5x} \end{aligned} \right\} \quad \frac{d}{dx} e^{ax} = ae^{ax}$$

$$y''_c - 7y'_c + 10y_c \stackrel{?}{=} 0$$

$$(4c_1 e^{2x} + 25c_2 e^{5x}) - 7(2c_1 e^{2x} + 5c_2 e^{5x}) + 10(c_1 e^{2x} + c_2 e^{5x}) \stackrel{?}{=} 0$$

$$4c_1 e^{2x} + 25c_2 e^{5x} - 14c_1 e^{2x} - 35c_2 e^{5x} + 10c_1 e^{2x} + 10c_2 e^{5x} \stackrel{?}{=} 0$$

$$-10c_1e^{2x} - 10c_2e^{5x} + 10c_1e^{2x} + 10c_2e^{5x} \stackrel{?}{=} 0$$
$$0 \stackrel{\checkmark}{=} 0$$

(2) Show y_p is a soln. of $y'' - 7y' + 10y = 24e^x$:

$$y_p = 6e^x$$

$$y'_p = 6e^x$$

$$y''_p = 6e^x$$

$$y''_p - 7y'_p + 10y_p \stackrel{?}{=} 24e^x$$

$$6e^x - 7(6e^x) + 10(6e^x) \stackrel{?}{=} 24e^x$$

$$6e^x - 42e^x + 60e^x \stackrel{?}{=} 24e^x$$

$$24e^x \stackrel{\checkmark}{=} 24e^x$$

(3) Show $y_c + y_p$ is a soln. of $y'' - 7y' + 10y = 24e^x$:

$$y_c + y_p$$

$$(y_c + y_p)'$$

$$(y_c + y_p)''$$

$$(y_c + y_p)'' - 7(y_c + y_p)' + 10(y_c + y_p) \stackrel{?}{=} 24e^x$$

$$y_c'' + y_p'' - 7y_c' - 7y_p' + 10y_c + 10y_p \stackrel{?}{=} 24e^x$$

$$\underbrace{(y_c'' - 7y_c' + 10y_c)}_{\text{showed } = 0} + \underbrace{(y_p'' - 7y_p' + 10y_p)}_{\text{showed } = 24e^x} \stackrel{?}{=} 24e^x$$

in #①

in #②

$$0 + 24e^x \checkmark \stackrel{?}{=} 24e^x$$