

Lecture

Section 5.5 The Substitution Rule.

This rule handles some difficult to integrate functions (that can usually be viewed as the composition of two functions multiplied by another function). It is the CHAIN RULE in reverse.

FORMULA

$$\int g'(x) f(g(x)) dx = F(g(x)) + C$$

where $F'(x) = f(x)$ (and so

$$\frac{d}{dx} F(g(x)) = F'(g(x)) \cdot g'(x) = f(g(x)) \cdot g'(x)$$

To handle integrals like

$$\int g'(x) f(g(x)) dx$$

we will not use the FORMULA but instead do the following:

let $u = g(x)$. Then $\frac{du}{dx} = g'(x)$.

Substitute u for $g(x)$ and $\frac{du}{dx}$ for $g'(x)$ to obtain a new, easier to integrate, integral

$$\int \frac{du}{dx} f(u) dx = \int f(u) \frac{du}{dx} dx$$

Treat $\frac{du}{dx}$ like
a fraction and
cancel the dx 's

$$= \int f(u) du$$

$$= F(u) + C$$

Substitute back
the $g(x)$ for the u

$$= F(g(x)) + C$$

Examples.

1. $\int 2x e^{x^2} dx = ?$

$$\int \begin{matrix} 2x \\ g'(x) \end{matrix} e^{\begin{matrix} x^2 \\ g(x) \end{matrix}} dx$$

let $u = x^2$. Then $\frac{du}{dx} = 2x$. So

$$\int 2x e^{x^2} dx = \int \frac{du}{dx} e^u dx$$

$$= \int e^u \frac{du}{dx} dx$$

$$= \int e^u du$$

$$= e^u + C$$

$$\boxed{e^{x^2} + C}$$

2. $\int 3x^2(x^3+1)^{10} dx = ?$

$$\int \underbrace{3x^2}_{g'(x)} \underbrace{(x^3+1)^{10}}_{g(x)} dx$$

Let $u = x^3 + 1$. Then $\frac{du}{dx} = 3x^2$. So

$$\int 3x^2(x^3+1)^{10} dx = \int \frac{du}{dx} u^{10} dx$$

$$= \int u^{10} \frac{du}{dx} dx$$

$$= \int u^{10} du$$

$$= \frac{u^{11}}{11} + C$$

$$= \boxed{\frac{(x^3+1)^{11}}{11} + C}$$