Lecture

Section 2.5. Limits Involving Infinity.

"Limits involving infinity" fall into one of 3 categories:

$\boxed{\text{CATEGORY 1}}$ $\lim_{x \to a} f(x) = +\infty$ (or $-\infty$)

"As x approaches the value a, f(x) gets larger and larger ($+\infty$) (or more and more negative ($-\infty$)."

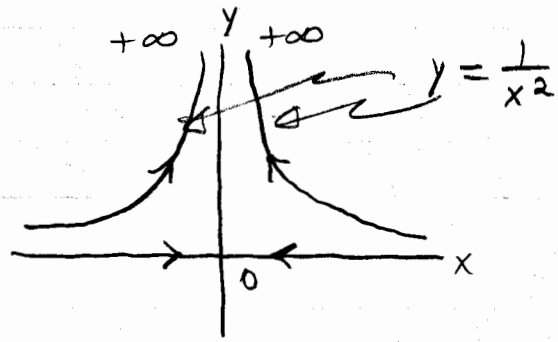
This limit really does <u>not</u> exist, but the above notation shows <u>how</u> it does not exist.

Use TABLES or GRAPHS to evaluate this kind of limit. I will use graphs.

Examples.

1. $\lim_{x \to 0} \frac{1}{x^2} = ?$

By calculator or by hand, the graph of $f(x) = \frac{1}{x^2}$ is



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$\therefore \lim_{x \to 0^-} \frac{1}{x^2} = +\infty$, $\lim_{x \to 0^+} \frac{1}{x^2} = +\infty \Rightarrow$

right and left hand limits are equal and they equal $+\infty$

$$\lim_{x \to 0} \frac{1}{x^2} = \boxed{+\infty}$$

2. $\lim_{x \to 1^-} \frac{1}{x-1} = ?$ and $\lim_{x \to 1^+} \frac{1}{x-1} = ?$

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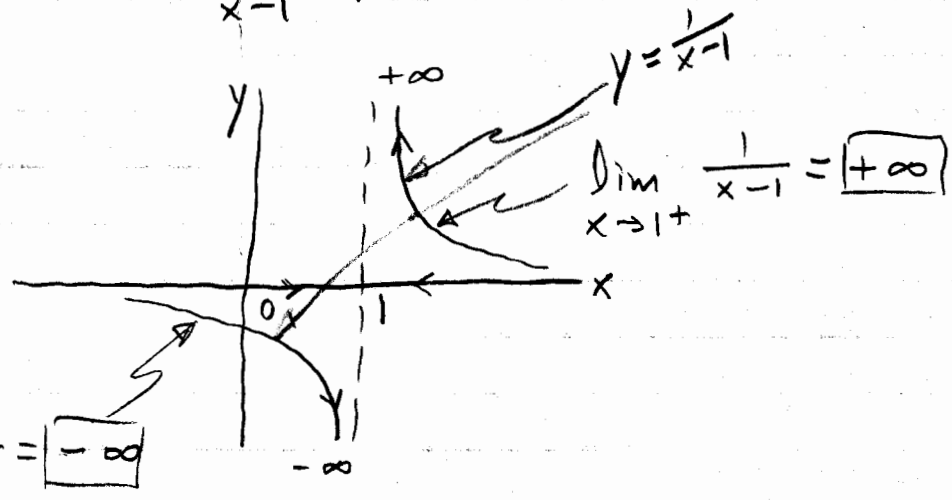
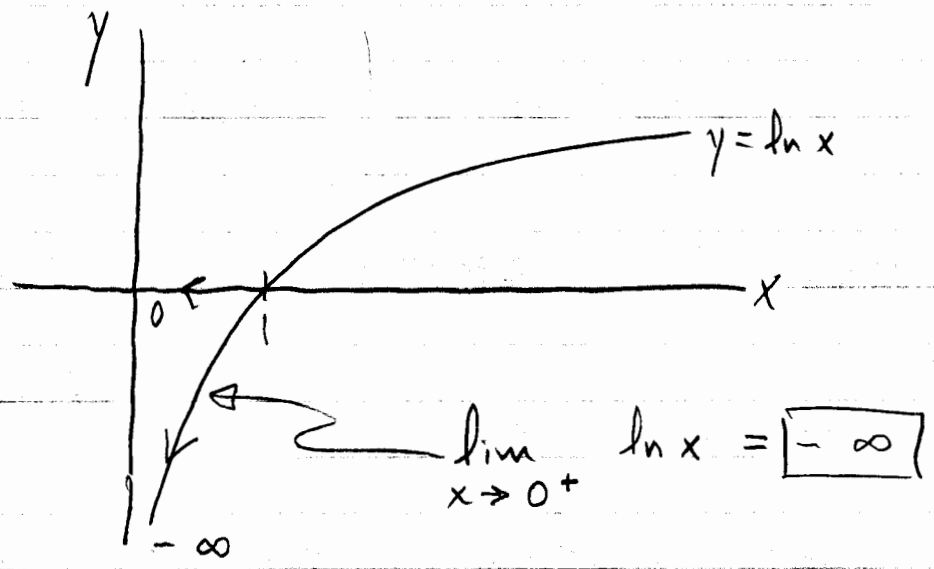
Graph of $f(x) = \frac{1}{x-1}$:



3. $\lim_{x \to 0^+} \ln x = ?$

Graph of $f(x) = \ln x \ (= \log_e x)$:



Calculator: xMin = 0.1
xMax = 10
yMin = −10
yMax = 5

But you have to know that the y-axis is a vertical asymptote and DOMAIN = $\{x \mid x > 0\}$!

$\boxed{\text{CATEGORY 2}}$ $\lim\limits_{x \to +\infty \text{ (or } -\infty)} f(x) = L$

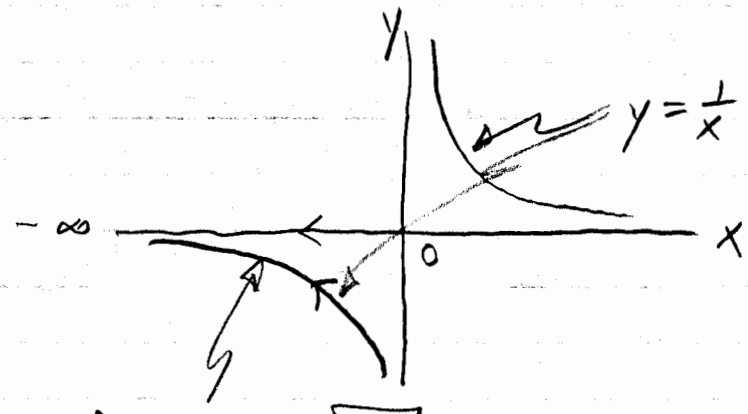
"As x gets larger and larger ($+\infty$) (or more and more negative ($-\infty$)), f(x) approaches the finite value L."

This limit <u>does</u> exist, since L is a single finite value.

<u>Examples</u>.

1. $\lim\limits_{x \to -\infty} \frac{1}{x} = ?$

Graph of $f(x) = \frac{1}{x}$:



$\lim\limits_{x \to -\infty} \frac{1}{x} = \boxed{0}$

2. (HW Exercise 20, p. 140.)

$$\lim_{t \to \infty} \frac{7t^3 + 4t}{2t^3 - t^2 + 3} = ?$$

We will do some ALGEBRA and use the fact that

For any pos. integer n and any constant c

$$\lim_{t \to \infty} \frac{c}{t^n} = 0 \quad \text{and} \quad \lim_{t \to -\infty} \frac{c}{t^n} = 0.$$

$$\lim_{t \to \infty} \frac{7t^3 + 4t}{2t^3 - t^2 + 3} = \lim_{t \to \infty} \frac{7t^3 + 4t}{2t^3 - t^2 + 3} \cdot \frac{\frac{1}{t^3}}{\frac{1}{t^3}}$$

$$= \lim_{t \to \infty} \frac{7\cancel{t^3} \cdot \frac{1}{\cancel{t^3}} + 4t \cdot \frac{1}{t^3}}{2\cancel{t^3} \cdot \frac{1}{\cancel{t^3}} - t^2 \cdot \frac{1}{t^3} + 3 \cdot \frac{1}{t^3}}$$

$$= \lim_{t \to \infty} \frac{7 + \underbrace{\frac{4}{t^2}}_{\to 0}}{2 - \underbrace{\frac{1}{t}}_{\to 0} + \underbrace{\frac{3}{t^3}}_{\to 0}} = \frac{7 + 0}{2 - 0 + 0}$$

$$= \boxed{\frac{7}{2}}$$

$\boxed{\text{CATEGORY 3}}$ $\lim\limits_{x \to +\infty \text{ (or } -\infty)} f(x) = +\infty$ (or $-\infty$)

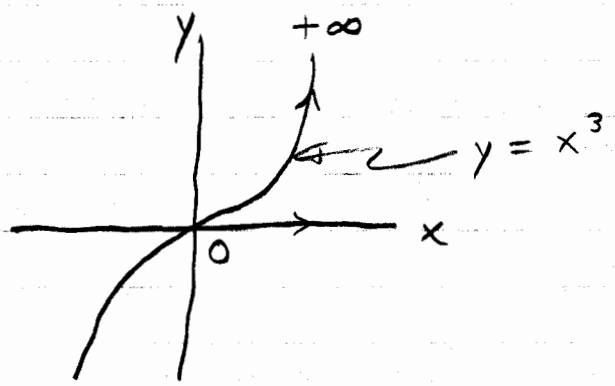
"As x gets larger and larger (or more and more negative, so does $f(x)$."

This limit does <u>not</u> exist.

<u>Examples</u>.

1. $\lim\limits_{x \to +\infty} x^3 = ?$

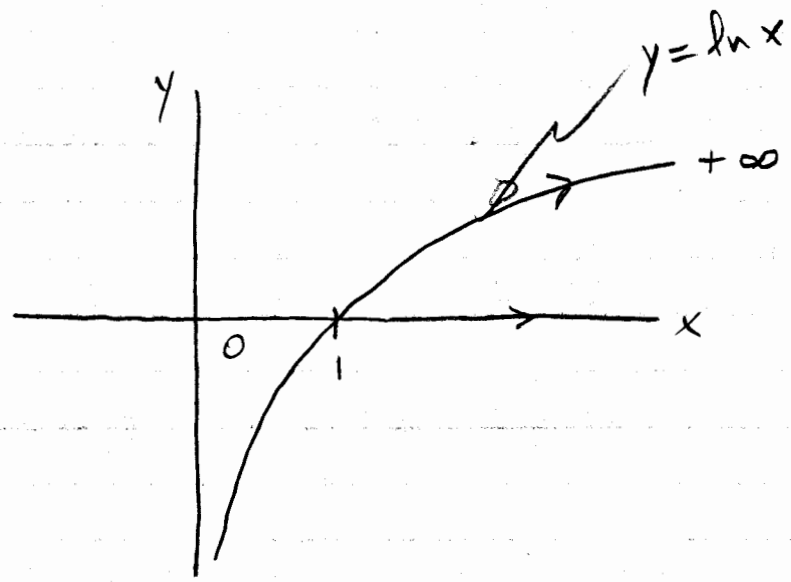
Graph of $f(x) = x^3$:



$$\lim_{x \to +\infty} x^3 = \boxed{+\infty}$$

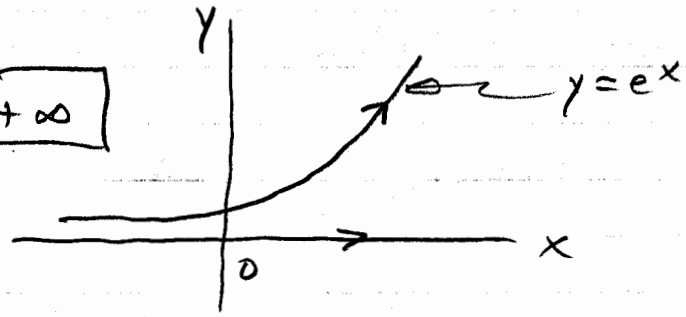
2. $\lim_{x \to +\infty} \ln x = ?$

Graph of $f(x) = \ln x$:



$$\lim_{x \to +\infty} \ln x = \boxed{+\infty}$$

3. $\lim_{x \to +\infty} e^x = ?$

Graph of $f(x) = e^x$:

$$\lim_{x \to +\infty} e^x = \boxed{+\infty}$$