How can we use the computer as a tool for investigating graph-theoretic independence? Here are our goals for Sage/CoCalc.

1. Graphs
   (a) Built-in graphs. Many common graphs and graph classes are built-in to Sage. How do we access and use them?
   (b) Building graphs. How can I build my own graph?

2. Graph Representations
   (a) How can I get the adjacency matrix for a graph?
   (b) How can I build a graph with a given adjacency matrix?
   (c) How can I build a graph from other standard graph representations?

3. Graph Independence
   (a) How can we get a maximum independent set?
   (b) How can we get the independence number?

4. Graph Invariants.
   (a) An invariant is a number associated with a graph (or a function which returns a number); it may be an integer or a real number, etc.
   (b) What invariants are built in?
   (c) How can we code our own invariants?

5. Graph Properties
   (a) A property is a boolean (or a function which returns a boolean): True or False.
   (b) What properties are built in?
   (c) How can we code our own properties?

6. Searching Graphs
   (a) How can I generate all graphs with some order (or with some extra conditions)? (nauty).
   (b) How can I generate random graphs of some order (with different models, or edge probabilities)?

7. Graph Algorithms
   (a) We’ll code the naive algorithm.
   (b) We’ll code the Tarjan-Trojanowski algorithm (and see an example of a recursive algorithm).
8. Integer and Linear Programming

(a) How can I solve integer and linear programs?
(b) How can I add constraints representing graph theory problems (especially edge constraints)?
(c) How can I find the fractional independence number?

9. Critical Independent Sets

(a) How can we find a maximum critical independent set in a graph?
(b) How can we find the sets $X$ and $X^c$ from the Independence Decomposition Theorem?
(c) How can we test if a graph is independence irreducible?
(d) How

10. The CONJECTURING Program and $\texttt{gt.sage}$.

(a) How can we generate upper or lower bounds for the independence number of a graph?
(b) How can we access the many graphs and invariants that have been stored in the file $\texttt{gt.sage}$?

11. Python, Files, Programming, Timing

(a) We’ll see basic Python programming structures (for loops, etc).
(b) How can we store and re-use larger pieces of code.
(c) How can we test the speed of our algorithms?