Last name _____

First name _____

LARSON—MATH 756—HOMEWORK WORKSHEET 11
Test 2 Review

Turn in a nicely written-up test review at test time.

Definitions. Give the definition and an example illustrating each definition.

1. residue.
2. annihilation number.
3. Cvetkovic bound.
4. chordal graph.
5. simplicial vertex elimination ordering.
6. odd hole.
7. perfect graph.
8. α-critical graph.

Theorems. State each theorem carefully.

9. Havel-Hakimi Theorem.
10. Residue and Annihilation.
11. Interlacing Theorem.
12. Cvetkovic's Theorem.
13. Perfect Graph Theorem.

Proofs. Give careful and complete proofs of the following.

14. Prove that the eigenvalues of a real symmetric matrix are real.
15. Prove that (the first part of) the Interlacing Theorem implies the first half of Cvetkovic's Theorem: that is, prove that the independence number of a graph is no more than the number of non-negative eigenvalues.
16. Prove that the independence number of a graph is no more than its annihilation number.

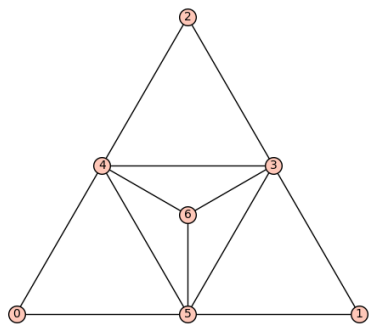
Algorithms

17. Describe an *efficient* algorithm to find a maximum critical independent set in a graph.

Problems Explain everything.

18. Let U, W be 2-dimensional subspaces of $\mathbb{R}^3$. Argue that there is a non-zero vector in $U \cap W$.

19. Find the *Cvetkovic bound* for p_3.



20. Find the residue of the pyramid graph.

21. Find the annihilation number of the pyramid graph.

22. Show that every induced subgraph of a chordal graph is chordal.

23. Show that a graph is chordal if and only if every cycle with more than three vertices has a chord.

24. Find a simplicial vertex elimination ordering of the pyramid graph.

25. Show that bipartite graphs are perfect.

26. Find an example of a perfect graph which is not König-Egervary.

27. Find an example of a König-Egervary graph which is not perfect.

28. Let τ denote the cardinality of a minimum vertex cover (the smallest number of vertices needed so that each edge of the graph is incident to at least one of these vertices). A graph G is τ-critical if for every edge xy, $\tau(G - xy) < \tau(G)$. Show a connected graph is τ-critical if and only if it is alpha-critical.

29. Let G be the pyramid graph. Find an α-critical subgraph H of G with $\alpha(H) = \alpha(G)$.

Bonus

30. Suppose that a graph is independence irreducible (which we now know means that the all-$\frac{1}{2}$'s solution is an optimal solution to the FLIP). Argue that, for every optimal solution $\{x_v : v \in V\}$, and edge uw, that $x_u + x_w = 1$ (that is, the sum of the weights of the endpoints of every edge must equal 1 for *any* optimal solution).