Suppose $G$ is a graph with vertices $V = V(G)$, and $\{x_i\}$ is an optimal FILP solution with $\alpha_f(G) = \sum x_i$. Let $\mathcal{O} = \{v_i : x_i = 1\}$, and $v \in \mathcal{O}$.

1. Show: $\alpha_f(G) = \alpha_f(G - v - N(v)) + 1$.

2. Describe an efficient algorithm for finding a maximum cardinality critical independent set (MCIS) in a graph (you can assume that solving LP’s is efficient).

3. Apply your algorithm to find an MCIS in the king graph. (Of course don’t solve any LPs—use theory to claim an optimum on each iteration). Describe all the steps.

4. Describe an efficient algorithm for finding the Independence Decomposition of a graph.

5. Apply your algorithm to the king graph. Explain.