Last name _____

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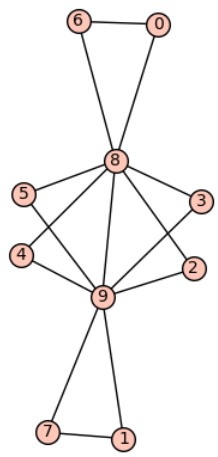
LARSON—MATH 756—CLASSROOM WORKSHEET 20
Chordal Graphs.

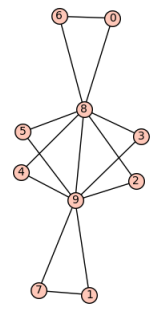


1. Can you *prove* the above graph is chordal?

2. Start with an empty set I. Find a simplicial vertex v in the above graph. Put v in I and remove its neighbors. Repeat. Stop when no vertices remain. Check that you have a maximum independent set.

A *simplicial vertex elimination ordering* of a graph G with order n is a listing of the vertices $v_1, v_2, \ldots, v_n$ such that, for each $i \in \{i, i+1, \ldots, n\}$, v_i is a simplicial vertex in the graph $G[\{v_i, v_{i+1}, \ldots, v_n\}]$.

3. Find a simplicial vertex elimination ordering for the above graph.



Greedy Coloring Algorithm: Reverse the simplicial ordering and consider the graphs induced on the last vertex, then the last two vertices, etc. In general, let $G_i = G[\{v_{(n-i)+1}, \ldots, v_n\}]$, for $i \in \{1, 2. \ldots, n\}$. So $G_1 = G[\{v_n\}]$, $G_2 = G[\{v_{n-1}, v_n\}]$,..., $G_n = G[\{v_1, v_2, \ldots, v_n\}]$. Color vertex v_n in G_1 with color 1, color vertex v_{n-1} in G_2 with color 2, color vertex v_{n-2} in G_3 with the smallest available color of 1 and 2 or, if v_{n-2} is adjacent to both v_{n-1} and v_n then use a new color (integer). In general color vertex v_i in $G_{(n-i)+1}$ with the smallest available color, or if v_i is adjacent to vertices using all previously used colors choose a new color.

4. Reverse your simplicial ordering.

5. Draw the graphs $G_1, \ldots G_{10}$ as defined above.

6. Color each graph according to the Greedy Coloring Algorithm.

7. For each graph G_i, check that $\omega(G_i) = \chi(G_i)$.