Last name _____

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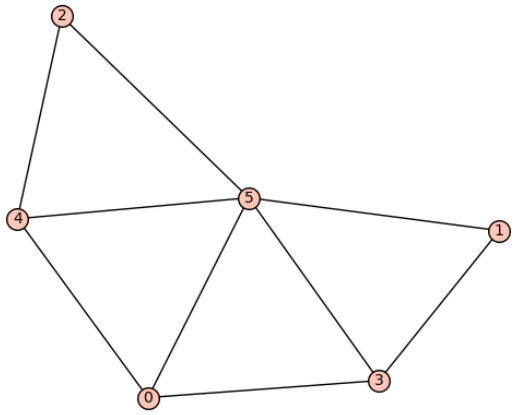
LARSON—MATH 756—CLASSROOM WORKSHEET 19
Chordal Graphs.



A vertex v is a *simplicial vertex* if $N[v] = N(v) \cup \{v\}$ is (induces) a clique.

1. Find all the simplicial vertices in the above graph.

2. Find α. Choose a simplicial vertex v and find a maximum independent set containing v.

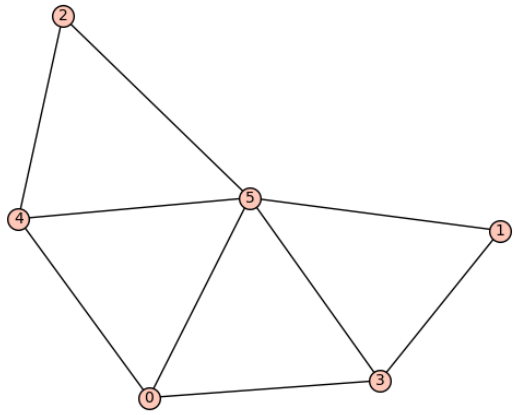
A graph is *chordal* if the only induced cycles are triangles.

3. Is this graph chordal? (Can you *prove* it?)

If C is a cycle in a graph containing vertices x and y that are not consecutive on the cycle, but are adjacent, then edge xy is a *chord* of G.

4. Prove: if a graph is chordal then every cycle with more than three vertices has a chord.

5. Prove: if every cycle with more than three vertices has a chord then the graph is chordal.



6. A graph property P is *hereditary* if, for any graph G with property P and any subset $S \subset V(G)$, the induced subgraph $G[S]$ has property P.

7. Prove: if a graph is chordal then every induced subgraph is also chordal (and thus being chordal is a hereditary property).

If x and y are non-adjacent vertices in a graph G, an *x-y-cut* is a cut $S \subseteq V(G)$ such that x and y are in different components of $G[V \setminus S]$.

8. Explain why, for non-adjacent vertices x and y, that there *is* an x-y-cut.

An x-y-cut S is *minimal* if no proper subset of S is an x-y-cut.

9. Find a minimal 1-2-cut in the above graph.

10. Explain why, for non-adjacent vertices x and y, that there *is* a minimal x-y-cut.

A *simplicial vertex elimination ordering* of a graph G with order n is a listing of the vertices $v_1, v_2, \ldots, v_n$ such that, for each $i \in \{i, i+1, \ldots, n$, v_i is a simplicial vertex in the graph $G[\{v_i, v_{i+1}, \ldots, v_n\}]$.

11. Find a simplicial vertex elimination ordering for the above graph.