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LARSON—MATH 756—CLASSROOM WORKSHEET 17
Independence Number of a Graph—Cvetkovic's Theorem.

The **Cvetvovic bound** of a graph G is the minimum of the number of non-negative and non-positive eigenvalues of the adjacency matrix of the graph.

Notation: Let ρ_0 be the number of 0 eigenvalues, ρ_+ be the number of positive eigenvalues, and ρ_- be the number of negative eigenvalues. Then **Cvetkovic's Theorem** says $\alpha \leq \rho_0 + \min\{\rho_+, \rho_-\}$.

1. The eigenvalues of p_3 are $\sqrt{2}, -\sqrt{2}, 0$. Find an eigenvector corresponding to each, and check that they are mutually orthogonal (and hence linearly independent, and hence a basis for $\mathbb{R}^3$).

2. Check Cvetkovic's Theorem for c_4.

3. Let U, W be 2-dimensional subspaces of $\mathbb{R}^3$. Argue that there is a non-zero vector in $U \cap W$.