Independence Number of a Graph—Cvetkovic’s Theorem.

The Cvetvovic bound of a graph $G$ is the minimum of the number of non-negative and non-positive eigenvalues of the adjacency matrix of the graph.

**Notation:** Let $\rho_0$ be the number of 0 eigenvalues, $\rho_+$ be the number of positive eigenvalues, and $\rho_-$ be the number of negative eigenvalues. Then **Cvetkovic’s Theorem** says $\alpha \leq \rho_0 + \min\{\rho_+, \rho_-\}$.

1. The eigenvalues of $p_3$ are $\sqrt{2}, -\sqrt{2}, 0$. Find an eigenvector corresponding to each, and check that they are mutually orthogonal (and hence linearly independent, and hence a basis for $\mathbb{R}^3$).

2. Check Cvetkovic’s Theorem for $c_4$. 
3. Let $U, W$ be 2-dimensional subspaces of $\mathbb{R}^3$. Argue that there is a non-zero vector in $U \cap W$. 