Last name _____

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LARSON—MATH 756—CLASSROOM WORKSHEET 15
Independence Number of a Graph—Cvetkovic's Theorem.

The **Cvetvovic bound** of a graph G is the minimum of the number of non-negative and non-positive eigenvalues of the adjacency matrix of the graph.

Notation: Let ρ_0 be the number of 0 eigenvalues, ρ_+ be the number of positive eigenvalues, and ρ_- be the number of negative eigenvalues. Then **Cvetkovic's Theorem** says $\alpha \leq \rho_0 + \min\{\rho_+, \rho_-\}$.

1. Find the eigenvalues of k_2.

2. Check Cvetkovic's Theorem for k_2.

3. Find the eigenvalues of p_3.

4. Check Cvetkovic's Theorem for p_3.

5. Find the eigenvalues of k_3.

6. Check Cvetkovic's Theorem for k_3.

7. The *spectrum* (set of eigenvalues) of the Petersen graph is: $3^1, 1^5, -2^4$. Check Cvetkovic's Theorem for the Petersen graph.

8. Let U, W be 2-dimensional subspaces of $\mathbb{R}^3$. Argue that there is a non-zero vector in $U \cap W$.