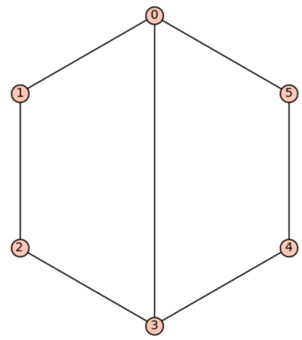
Last name ______________________

First name ______________________

LARSON—MATH 756—CLASSROOM WORKSHEET 14
Degree Bounds for the Independence Number

The *degree sequence* (d) of a graph of order n is a list of the degrees of its vertices, usually written in non-increasing order. So $(d) = (d_1, d_2, \ldots, d_n)$ with $d_1 \geq d_2 \geq \ldots \geq d_n$).



1. Find the degree sequence for the ("squares") graph above.

A *graphic sequence* is a list of non-negative integers that is the degree sequence of some graph. A graph with degree sequence (d) *realizes* (d).

2. Is the sequence $(d) = (3, 3, 3, 3, 3, 2, 2, 1)$ graphic?

3. Is the sequence $(d) = (6, 3, 3, 3, 3, 3, 3, 3, 1)$ graphic?

4. Is the sequence $(d) = (2, 2, 2, 1, 1, 1, 1, 1, 1)$ graphic?

Given a non-increasing sequence $(d) = (d_1, d_2, \ldots, d_n)$, the Havel-Hakimi *derived sequence* (d') is the sequence formed by (1) deleting d_1, (2) reducing the next d_1 terms of (d) by 1, and (3) reordering so that the sequence is non-increasing.

5. Let $(d) = (3, 3, 3, 3, 3, 2, 2, 1)$. Find (d').

6. How can you use the Havel-Hakimi Theorem to test if $(d) = (3, 3, 3, 3, 3, 2, 2, 1)$ is graphic?

7. Determine if the sequence $(d) = (6, 3, 3, 3, 3, 3, 3, 3, 1)$ is graphic.

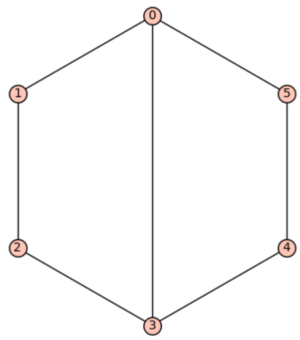
8. Determine if the sequence $(d) = (2, 2, 2, 1, 1, 1, 1, 1, 1)$ is graphic.

9. Find two non-isomorphic graphs with the degree sequence $(d) = (2, 2, 2, 1, 1, 1, 1, 1, 1)$.

The *Havel-Hakimi switch* (2-switch) is the operation of taking edges xy and vw in a graph where neither endpoint of xy is adjacent to either endpoint of vw, deleting those edges and adding edges xv and yw. Note that this operation does not change the degree sequence of the graph.

The Havel-Hakimi equivalence theorem says that, for any pair of graphs G and G' with the same degree sequence, there is a sequence of 2-switches that will transform G to G' (Berge, 1973).

10. Find a sequence of Havel-Hakimi switches that transforms your graphs in Problem **??** from one to the other.



Given a degree sequence (d) of a graph with order n now written in nondecreasing order—so $(d) = (d_1, d_2, \ldots, d_n)$ with $d_1 \leq d_2 \leq \ldots \leq d_n$), define the *annihilation number* a of G to be the largest index k such that $\sum_{i=1}^{k} \leq \sum_{i=k+1}^{n}$.

11. Find the annihilation number of the "squares" graph.

12. Find the annihilation number of a cycle C_n.

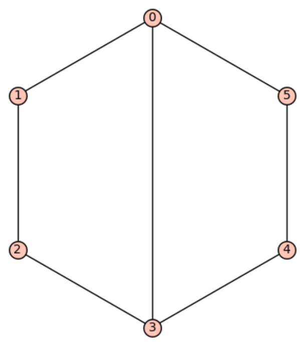
13. Find the annihilation number of a complete graph K_n.

The *Annihilation Bound Theorem* says: for any graph $a \geq \alpha$ (Fajtlowicz, Pepper)

14. Check the Annihilation Bound Theorem for the "squares" graph at the beginning of the worksheet.

15. Check the Annihilation Bound Theorem for the cycles C_n.

16. Check the Annihilation Bound Theorem for the complete graphs K_n.



The *residue* R of a graph G with (non-increasing) degree sequence (d) is the number of zeros after successive repetition of finding Havel-Hakimi derived sequences until you get an all-zeros sequence (the *Havel-Hakimi process*).

17. Find the residue of the "squares" graph.

18. Can you find a formula for the residue of a cycle C_n?

19. Can you find a formula for the residue of a complete graph K_n?

Fajtlowicz's GRAFFITI program conjectured that, for any graph, $\alpha \geq R$. (This was proved by Favoron, Maheo and Sacle). Call this the *Residue Bound Theorem.*

20. Check the Residue Bound Theorem for the "squares" graph.

21. Check the Residue Bound Theorem for the cycles C_n.

22. Check the Residue Bound Theorem for the complete graphs K_n.