For a connected graph, the *distance* $d(v, w)$ from vertex $v$ to vertex $w$ is the length (number of edges) in a shortest path from $v$ to $w$. The *eccentricity* $e(v)$ of a vertex $v$ is the maximum distance to any other vertex; formally, this is $e(v) = \max\{d(v, w) : w \in V\}$. The *radius* $r$ of a (connected) graph is the minimum eccentricity of any of its vertices.

1. Find the radius for each of the following graphs.

2. In each graph above find an induced path with at least $2r - 1$ vertices.

3. In each graph above find the independence number $\alpha$ and check that $\alpha \geq r$.

A vertex $v$ in a graph $G$ is a *cut vertex* if the graph formed by removing $v$ and its incident edges increases the number of components of the graph.

4. Argue that any non-trivial connected graph has at least two non-cut vertices (*connectors*).
5. Show that each of the following graphs is bipartite.

6. Find the matching number $\nu$ for each of the above graphs.

7. Check that König’s Theorem holds for each of the above graphs.