
2. **NP-Completeness of MIS.** MIS (or equivalently Maximum Clique or Minimum Cover) was one of the original 21 problems that Karp proved is NP-complete in 1972 [21].

3. **Tarjan-Trojanowski Algorithm.** Tarjan and Trojanowski’s 1977 algorithm is in [39]. The state-of-the-art practical algorithm is Cliqueur due to Ostergard [33].

4. **Induced Path Theorem.** The theorem is in [16], and the proof there is credited to Fan Chung. This gives the radius lower bound \( \alpha \geq r \). Another proof of this theorem and a nice structural result is in [18].

5. **Lovasz’ Theta.** The original Lovasz paper is [29]. Knuth wrote an interesting 1991 paper with several equivalent definitions [22].

6. **Matching, König’s Theorem, Hungarian Method.** Berge’s 1957 theorem is in [4]. The Hungarian Method was first presented by Kuhn in [24]. The history of König’s Theorem, as well as proofs of several interesting equivalent theorems is in Lovasz and Plummer’s classic book on matching theory [30].

7. **König-Egervary Graphs.** The original papers were due to Deming and Sterboul, both in 1979 [12] [38].

8. **Integer Programming and Fractional Independence.** Balinki’s Lemma is usually cited as [3]. The Nemhouser-Trotter Theorem is in [32]. The Picard-Queyranne Theorem is in [35]. The Independence Decomposition Theorem is in [26].

9. **Critical Independent Sets.** These were first introduced by C.Q. Zhang in [40]. Ageev made the first connection between critical independent sets and linear programming in [1]. In 2007 Butheko [7] asked

   The problem of characterizing the graphs with nonempty critical independent sets in general is an interesting problem for future research. Since a critical weighted independent set is a part of a maximum weight independent set, another relevant question is, how to find the largest critical weighted independent set in a graph?

   Algorithms were reported in [25] and [11].

10. **Degree-Sequence Bounds.** Fajtlowicz introduced residue in [17] and the Graffiti conjecture that it is a lower bound for the independence number. The first proof was due to Favaron-Maheo-Saclé [19]. The shortest proof is likely Grigg’s and Kleitman’s [20]. The annihilation number was introduced by Pepper [34]. The characterization of graphs with equal annihilation and independence numbers is in [27].
11. **Eigenvalue Bounds.** Cvetkovic proved his 1971 eigenvalues upper bound for the independence number in [9]. The main tool is the super-useful Cauchy Interlacing Theorem (where and when is this from?). A very useful compilation of this and other graph eigenvalue results is [10]

12. **Chordal and Perfect Graphs.** The characterization theorem that a graph is chordal if and only if it has a simplicial vertex ordering is due to Dirac in 1961 [14]. The Perfect Graph Conjecture is due to Berge at the same time [37]. The Weak Perfect Graph Theorem was proved by Lovasz in 1972 [31]. The Strong Perfect Graph Theorem was proved by Chudnovsky, Robertson, Seymour and Thomas [8]. A polynomial-time recognition algorithm was proved shortly after [7].

13. **Cutting Planes & Alpha-Critical Graphs.** The literature on stable set (or vertex packing) polytopes is vast. Start with this comprehensive and useful reference filled with historical information.

Critical graphs were first studied by Dirac [13]. Lovasz reports that α-critical graphs were studied by Erdős and Gallai in the early 1960s (he cites [15]). The theorem that incident edges are contained in an induced odd cycle is due to Andrasfai [2]. Lovasz writes about α-critical graphs in many places. His first substantial results are in [28]

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**References**


