LARSON—MATH 756—Fall 2019
Annotated Bibliography (Dec. 1)

1. **Graph Theory Books**. The first graph theory book was published in German in 1939. An english translation [23] published in 1990. The first french graph theory book was Berge's in 1958 [4]. A still-influential english text is Bondy and Murty's from 1976 [6]. For an interesting history of graph theory see [5].

2. **NP-Completeness of MIS**. MIS (or equivalently Maximum Clique or Minimum Cover) was one of the original 21 problems that Karp proved is NP-complete in 1972 [21].

3. **Tarjan-Trojanowski Algorithm**. Tarjan and Trojanowski's 1977 algorithm is in [39]. The state-of-the art practical algorithm is `Cliquer` due to Ostergard [33].

4. **Induced Path Theorem**. The theorem is in [16], and the proof there is credited to Fan Chung. This gives the radius lower bound $\alpha \geq r$. Another proof of this theorem and a nice structural result is in [18].

5. **Lovasz' Theta**. The original Lovasz paper is [29]. Knuth wrote an interesting 1991 paper with several equivalent definitions [22].

6. **Matching, König's Theorem, Hungarian Method**. Berge's 1957 theorem is in [4]. The Hungarian Method was first presented by Kuhn in [24]. The history of König's Theorem, as well as proofs of several interesting equivalent theorems is in Lovasz and Plummer's classic book on matching theory [30].

7. **König-Egervary Graphs**. The original papers were due to Deming and Sterboul, both in 1979 [12] [38].

8. **Integer Programming and Fractional Independence**. Balinki's Lemma is usually cited as [3]. The Nemhouser-Trotter Theorem is in [32]. The Picard-Queyranne Theorem is in [35]. The Independence Decomposition Theorem is in [26].

9. **Critical Independent Sets**. These were first introduced by C.Q. Zhang in [40]. Ageev made the first connection between critical independent sets and linear programming in [1]. In 2007 Butheko [7] asked

   > The problem of characterizing the graphs with nonempty critical independent sets in general is an interesting problem for future research. Since a critical weighted independent set is a part of a maximum weight independent set, another relevant question is, how to find the largest critical weighted independent set in a graph?

   Algorithms were reported in [25] and [11].

10. **Degree-Sequence Bounds**. Fajtlowicz introduced *residue* in [17] and the Graffiti conjecture that it is a lower bound for the independence number. The first proof was due to Favaron-Maheo-Saclé [19]. The shortest proof is likely Grigg's and Kleitman's [20]. The *annihilation number* was introduced by Pepper [34]. The characterization of graphs with equal annihilation and independence numbers is in [27].

11. **Eigenvalue Bounds**. Cvetkovic proved his 1971 eigenvalues upper bound for the independence number in [9]. The main tool is the super-useful Cauchy Interlacing Theorem (where and when is this from?). A very useful compilation of this and other graph eigenvalue results is [10]

12. **Chordal and Perfect Graphs**. The characterization theorem that a graph is chordal if and only if it has a simplicial vertex ordering is due to Dirac in 1961 [14]. The Perfect Graph Conjecture is due to Berge at the same time [37]. The Weak Perfect Graph Theorem was proved by Lovasz in 1972 [31]. The Strong Perfect Graph Theorem was proved by Chudnovsky, Robertson, Seymour and Thomas [8]. A polynomial-time recognition algorithm was proved shortly after [**?**].

13. **Cutting Planes & Alpha-Critical Graphs**. The literature on stable set (or vertex packing) polytopes is vast. Start with this [36] comprehensive and useful reference filled with historical information.

    Critical graphs were first studied by Dirac [13]. Lovasz reports that α-critical graphs were studied by Erdős and Gallai in the early 1960s (he cites [15]). The theorem that incident edges are contained in an induced odd cycle is due to Andrasfai [2]. Lovasz writes about α-critical graphs in many places. His first substantial results are in [28]

References

[1] A. A. Ageev. On finding critical independent and vertex sets. *SIAM J. Discrete Math.*, 7(2):293–295, 1994.

[2] B. Andrásfai. On critical graphs, theory of graphs (international symposium, rome, 1966), 1967.

[3] M. Balinski. On the maximum matching, minimum covering. In *Proc. Symp. Math. Programming*, pages 301–312. Princeton University Press, 1970.

[4] C. Berge. Two theorems in graph theory. *Proceedings of the National Academy of Sciences of the United States of America*, 43(9):842, 1957.

[5] N. Biggs, E. K. Lloyd, and R. J. Wilson. *Graph Theory, 1736-1936*. Clarendon Press, 1986.

[6] J. A. Bondy and U. S. R. Murty. *Graph theory with applications*. American Elsevier Publishing Co., Inc., New York, 1976.

[7] S. Butenko and S. Trukhanov. Using critical sets to solve the maximum independent set problem. *Oper. Res. Lett.*, 35(4):519–524, 2007.

[8] M. Chudnovsky, N. Robertson, P. Seymour, and R. Thomas. The strong perfect graph theorem. *Annals of mathematics*, pages 51–229, 2006.

[9] D. M. Cvetković. Graphs and their spectra. *Publikacije Elektrotehničkog fakulteta. Serija Matematika i fizika*, (354/356):1–50, 1971.

[10] D. M. Cvetković, M. Doob, and H. Sachs. *Spectra of graphs*. Johann Ambrosius Barth, Heidelberg, third edition, 1995. Theory and applications.

[11] E. DeLaVina and C. E. Larson. A parallel algorithm for computing the critical independence number and related sets. *Ars Mathematica Contemporanea*, 6:237–245, 2013.

[12] R. W. Deming. Independence numbers of graphs—an extension of the Koenig-Egervary theorem. *Discrete Math.*, 27(1):23–33, 1979.

[13] G. A. Dirac. The colouring of maps. *Journal of the London Mathematical Society*, 1(4):476–480, 1953.

[14] G. A. Dirac. On rigid circuit graphs. In *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, volume 25, pages 71–76. Springer, 1961.

[15] P. Erdos and T. Gallai. On the minimal number of vertices representing the edges of a graph. *Publ. Math. Inst. Hungar. Acad. Sci*, 6(18):1–203, 1961.

[16] P. Erdős, M. Saks, and V. T. Sós. Maximum induced trees in graphs. *J. Combin. Theory Ser. B*, 41(1):61–79, 1986.

[17] S. Fajtlowicz. Written on the wall: Conjectures of Graffiti. `http://math.uh.edu/~siemion`, 1986–2004.

[18] S. Fajtlowicz. A characterization of radius-critical graphs. *J. Graph Theory*, 12(4):529–532, 1988.

[19] O. Favaron, M. Mahéo, and J.-F. Saclé. On the residue of a graph. *J. Graph Theory*, 15(1):39–64, 1991.

[20] J. R. Griggs and D. J. Kleitman. Independence and the Havel-Hakimi residue. *Discrete Math.*, 127(1-3):209–212, 1994. Graph theory and applications (Hakone, 1990).

[21] R. M. Karp. Reducibility among combinatorial problems. In *Complexity of computer computations*, pages 85–103. Springer, 1972.

[22] D. E. Knuth. The sandwich theorem. *Electron. J. Combin.*, 1:Article 1, approx. 48 pp. (electronic), 1994.

[23] D. König. Theory of finite and infinite graphs. In *Theory of Finite and Infinite Graphs*, pages 45–421. Springer, 1990.

[24] H. W. Kuhn. The hungarian method for the assignment problem. *Naval research logistics quarterly*, 2(1-2):83–97, 1955.

[25] C. E. Larson. A note on critical independence reductions. *Bull. Inst. Combin. Appl.*, 51:34–46, 2007.

[26] C. E. Larson. The critical independence number and an independence decomposition. *European J. Combin.*, 32(2):294–300, 2011.

[27] C. E. Larson and R. Pepper. Graphs with equal independence and annihilation numbers. *Electronic Journal of Combinatorics*, 18(1), 2011.

[28] L Lovász. Some finite basis theorems in graph theory. *Combinatorics, Coll. Math. Soc. J. Bolyai*, 18:717–729, 1978.

[29] L. Lovász. On the Shannon capacity of a graph. *Information Theory, IEEE Transactions on*, 25(1):1–7, 1979.

[30] L. Lovász and M. D. Plummer. *Matching theory*, volume 121 of *North-Holland Mathematics Studies*. North-Holland Publishing Co., Amsterdam, 1986. Annals of Discrete Mathematics, 29.

[32] G. L. Nemhauser and L. E. Trotter, Jr. Vertex packings: structural properties and algorithms. *Math. Programming*, 8:232–248, 1975.

[33] P. R. J. Östergård. A fast algorithm for the maximum clique problem. *Discrete Appl. Math.*, 120(1-3):197–207, 2002. Sixth Twente Workshop on Graphs and Combinatorial Optimization (Enschede, 1999).

[34] R. Pepper. On the annihilation number of a graph. *Recent Advances in Applied Mathematics and Computational And Information Sciences*, 1:217–220, 2009.

[35] J.-C. Picard and M. Queyranne. On the integer-valued variables in the linear vertex packing problem. *Math. Programming*, 12(1):97–101, 1977.

[36] A. Schrijver. *Combinatorial optimization: polyhedra and efficiency*, volume 24. Springer Science & Business Media, 2003.

[37] P. Seymour. How the proof of the strong perfect graph conjecture was found. *Gazette des Mathematiciens*, 109:69–83, 2006.

[38] F. Sterboul. A characterization of the graphs in which the transversal number equals the matching number. *J. Combin. Theory Ser. B*, 27(2):228–229, 1979.

[39] R. E. Tarjan and A. E. Trojanowski. Finding a maximum independent set. *SIAM J. Comput.*, 6(3):537–546, 1977.

[40] C. Q. Zhang. Finding critical independent sets and critical vertex subsets are polynomial problems. *SIAM J. Discrete Math.*, 3(3):431–438, 1990.