LARSON—MATH 750—Course Outline
Posets & Mobius Inversion

1. **Definition**. An *independent set* in a graph is a set of vertices which are pair-wise non-adjacent. A *maximum independent set* (MIS) is a largest cardinality independent set. The *independence number* α is the cardinality of an MIS.

2. **A Hard Problem**. Calculating α in a general graph.

3. **A Different Problem**. Find a "good" approximation algorithm for α. It would be nice if the algorithm came with error bounds, etc.

4. **An Idea**: Consider some collection of subgraphs of a graph. If these define a poset we can use posetic ideas such as mobius inversion to define something like Taylor-approximating functions defined in terms of less complex graphs where computing the independence number is easier.

5. **Outline**.

   (a) **Basic Poset concepts and examples**. Comparability, incomparability, covering, Hasse diagram,

   (b) **Fundamental Poset Theorems**. Linear extension, Minty's Theorem, Dilworth's Theorem.

   (c) **Comparability graphs of posets**. Hoffman's characterization, perfection.

   (d) **Incidence algebras**. Zeta, Delta and existence of inverses.

   (e) **Mobius functions**. Fundamental Theorem, computation of paradigm examples.

   (f) **Product posets and mobius functions of them**. Application to Euler's totient function.

   (g) **Linear Algebra and posets**. Representation of incidence algebra operations by matrices.

   (h) **Graph posets**. Can we calculate mobius functions for various graph partial orders?

   (i) **Chemistry examples**. What has been done here, in connection to graphs and graph theory.

   (j) **Experiments**. Applications to graph invariant calculations, in particular the independence number of a graph.

6. **Goals**. New theorems, new insights into mobius approximation of graph invariants, new experimental results.