Last name ______________________

First name ______________________

LARSON—MATH 750—HOMEWORK WORKSHEET 08
Subspaces of Vector Spaces

Our goal is to compute the mobius function for the collection of subspaces of a vector space over a finite field, with the subspace relation. Our notation follows that of the Bender & Goldman article. Here is a small example to motivate their discussion.

Let $V_n(q)$ be the vector space consisting of vectors with n components and entries from the field $\mathbb{F}_q$.

Let $L(V_n(q))$ be the collection of subspaces of $V_n(q)$.

For $U, V \in L(V_n(q))$ define $U \leq V$ iff U is a subspace of V. We claim that $(L(V_n(q)), \leq)$ is a poset.

Let $\binom{n}{k}_q$ be the number of k-dimensional subspaces of $V_n(q)$.

This isn't a course on abstract or linear algebra—but the connections are many and we need to be facile with basic facts from these subjects. So first reload our vocabulary.

1. What is a field?
2. What is $\mathbb{F}_3$?
3. Show $\mathbb{F}_3$ is a field.
4. Write out all the vectors of $V_2(3)$.
5. What is a vector space?
6. Show that $V_2(3)$ is a vector space.
7. What is a subspace of a vector space?
8. Show that $(L(V_n(q)), \leq)$ is a poset.
9. What does it mean for a subspace of $V_n(q)$ to be 1-dimensional?
10. Find all 1-dimensional subspaces of $V_2(3)$.
11. Find $\binom{2}{1}_3$.
12. Find $\binom{2}{0}_3$.
13. Find $\binom{2}{2}_3$.
14. Give all of the subspaces of $V_2(3)$ names and draw a Hasse diagram.
15. Find a linear ordering of $(L(V_2(3)), \leq)$.

16. Make a table—use your linear ordering to index the rows and columns. For each row U and column V, find $\mu(U, V)$.

We will prove that the value of the mobius function $\mu_n = \mu(0, V_n(q))$ is given by:

$$\mu_n = (-1)^n \cdot q^{\binom{n}{2}}$$

where 0 denotes the unique 0-dimensional vector space.

17. Check that the value you calculated for $\mu(0, V_2(3))$ agrees with μ_2.