Last name ______________________

First name ______________________

LARSON—MATH 750—HOMEWORK WORKSHEET 07
Test 1 Review

Turn in a nicely written-up test review at test time.

Definitions. Give the definition and an example illustrating each definition.

1. A *partial order* on a set X.
2. The *Hasse diagram* of a poset.
3. *comparable* elements in a poset $\mathbb{P} = (X, \leq)$.
4. A *minimal* element in a poset $\mathbb{P} = (X, \leq)$.
5. An *anti-chain* in a poset.
6. A *linear* poset.
7. A *linear extension* of a poset $\mathbb{P} = (X, \leq)$.
8. The *comparability graph* of a poset $\mathbb{P} = (X, \leq)$.
9. A *perfect graph*.
10. The *height* of a poset.
11. The *width* of a poset.
12. The *zeta function* for a poset $\mathbb{P} = (X, \leq)$.
13. The *mobius function* for a poset $\mathbb{P} = (X, \leq)$..
14. For a poset $\mathbb{P} = (X, \leq)$ and functions $f : X \to \mathbb{R}$ and $g : X \to \mathbb{R}$ define the *convolution product* $f * g$.

Theorems. State each theorem carefully.

15. Sperner's Theorem.
16. Mirsky's Theorem.
17. Dilworth's Theorem.
18. The Weak Perfect Graph Theorem.

Proofs. Give careful and complete proofs of the following.

19. Prove that the comparability graph of a poset is perfect.

20. Prove: every poset admits a linear extension.

21. Prove: Dilworth's Theorem.

Algorithms

22. State an algorithm for finding a linear extension of a poset.

Problems Explain everything.

23. Let G be a graph and let X be the collection of connected subgraphs of G. For subgraphs H_1 and H_2 of G define a relation "$\leq$" as follows: $H_1 \leq H_2$ if and only if H_1 is a subgraph of H_2. Argue that this relation defines a poset $(X, \leq)$.

24. What elements cover the primes in $([n], |)$?

25. Suppose $\mathbb{P} = (X, \leq)$ is a poset. Let G be the comparability graph of $\mathbb{P}$. Let $\bar{G}$ be the complement of G. Is $\bar{G}$ the comparability graph of a poset?

26. Argue that the comparability graph of a poset is perfect.

27. Find a maximum anti-chain in the divisibility poset $\mathbb{D} = ([n], |)$. Explain.

28. Let $\mathbb{P} = (X, \leq)$ be a poset, and $f : X \to \mathbb{R}$. Let δ be the delta function for $\mathbb{P}$. Explain why $f * \delta = f$ (where $*$ is the convolution product).

29. Show that the zeta function of a poset has an inverse with respect to the convolution product (that is, show that there a function g with $\zeta * g = g * \zeta = \delta$).

30. Let $\mathbb{D} = ([10], |)$. Find a linear extension of this poset. Explain.

31. Let $\mathbb{D} = ([10], |)$ and Make a table with the sets in $[10]$ as the indices for the rows x and columns y with table entries $\mu(x, y)$.

32. Let $\mathbb{P}_1 = (X, \leq_1)$ with mobius function μ_1 and $\mathbb{P}_2 = (X, \leq_2)$ with mobius function μ_2 be posets. We can define a poset on $X \times Y = \{(x, y) : x \in X \land y \in Y\}$ by defining the relation:

$$(x_1, y_1) \leq (x_2, y_2) \text{ if and only if } x_1 \leq x_2 \text{ and } y_1 \leq y_2.$$

$\mathbb{P}$ is a poset and has a corresponding mobius function μ. Show that:

$$\mu((x_1, y_1), (x_2, y_2)) = \mu_1(x_1, x_2) \cdot \mu_2(y_1, y_2)).$$