Last name ______________________

First name ______________________

LARSON—MATH 750—HOMEWORK WORKSHEET 06
Mobius Inversion Theorem

Let $\mathbb{P} = (X, \leq)$ be a poset and let $F : X \to \mathbb{R}$, and define $G : X \to \mathbb{R}$ as follows: $G(x) = \sum_{\{y:y \geq x\}} F(y)$.

1. Prove:

$$F(x) = \sum_{\{y:y \geq x\}} \mu(x, y) \cdot G(y).$$

   This is another version of the Mobius Inversion Theorem. You should be able to more or less imitate the proof from class/Brualdi.

2. Two posets $\mathbb{P}_1 = (X, \leq_1)$ and $\mathbb{P}_2 = (Y, \leq_2)$ are isomorphic if there is a bijection $f : X \to Y$ such that $x \leq_1 x'$ if and only if $f(x) \leq_2 f(x')$.

   Let S be a set of n elements and $\mathbb{P} = (\mathcal{P}(S), \subseteq)$ be the poset of subsets of S with inclusion. Let $I = \{0, 1\}$ be the two element poset defined by $0 < 1$. Let I^n be the product of I with itself n times and the corresponding product order we defined in class for a product of 2 posets extended to a product of n posets.

   Show that $\mathbb{P}$ and I^n are isomorphic.