Today we’ll look at matrices associated with Mobius functions.

For a poset $\mathbb{P} = (X, \leq)$, and linear extension, $x_1 \leq' x_2 \leq' \ldots \leq' x_n$, with associated zeta $\zeta$ and mobius $\mu$ functions, let $Z$ be an $n \times n$ matrix defined by $Z_{i,j} = \zeta(x_i, x_j)$ and let let $M$ be an $n \times n$ matrix defined by $M_{i,j} = \mu(x_i, x_j)$.

1. Let $D_6 = \mathbb{P} = ([6], |)$. Find a linear extension $x_1 \leq x_2 \leq \ldots x_6$. Use this to find $Z$.

2. What things do you immediately notice about $Z$?

3. Find $Z^{-1}$.

4. Draw $D_6$ and use this to find $\mu(x_i, x_j)$ for each pair $(x_i, x_j)$.

5. Use your results to find $M$.

6. Does $M = Z^{-1}$?

7. Is this true for any poset $\mathbb{P} = (X, \leq)$ and associated matrices $Z$, $M$?

8. What is the matrix significance for the formula we found: $\sum_{x_i \leq x_k \leq x_j} \mu(x_i, x_k) = 0$?

9. Give a matrix proof of the Mobius Inversion Theorem.

10. Use the Mobius Inversion Theorem to find a representation for the Euler Totient Function.