

Last name _____

First name _____

LARSON—MATH 750—CLASSROOM WORKSHEET 16
Inclusion-Exclusion

Let $S = \{a, b, c, d, e, f\}$. $A_1 = \{a, b, c, d\}$, $A_2 = \{b, c, d, e\}$ and $A_3 = \{a, d, e\}$.
Let $\bar{A}_i = S \setminus A_i$.

1. Find $A_1 \cup A_2 \cup A_3$ and $|A_1 \cup A_2 \cup A_3|$.

2. Find $\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3$ and $|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3|$.

Let $X_3 = [3]$. Define $F : \mathcal{P}(X_3) \rightarrow \mathbb{R}$ as follows:

$$F(K) = |\{s \in S : s \in A_i \forall i \in [3] \setminus K \wedge s \notin A_j \forall j \in K\}|.$$

3. Find $F(\{1\})$.

4. Find $F(\{2\})$.

5. Find $F(\{3\})$.

6. Find $F(\{1, 2\})$.

7. Find $F(\{1, 3\})$.

8. Find $F(\{2, 3\})$.

9. Find $F(\emptyset)$.

10. Find $F(\{1, 2, 3\})$.

11. Is it true that $F(\{1, 2, 3\}) = |\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3|$? (If so, why?)

Let $G(K) = \sum_{L \subseteq K} F(L)$.

12. Find $G(\{1\})$.

13. Find $G(\{2\})$.

14. Find $G(\{3\})$.

15. Find $G(\{1, 2\})$.

16. Find $G(\{1, 3\})$.

17. Find $G(\{2, 3\})$.

18. Find $G(\{1, 2, 3\})$.

19. Find $G(\emptyset)$.

20. For each $K \subseteq [3]$ find $\bigcap_{i \notin K} A_i$ and check that $G(K) = |\bigcap_{i \notin K} A_i|$.