Last name ______________________

First name ______________________

LARSON—MATH 750—CLASSROOM WORKSHEET 10
Convolution, Zeta and Mobius

For the poset $\mathbb{P} = (X, \leq)$, let $\mathcal{F} = \{f : X \times X \to \mathbb{R}\}$. Then define the *Kronecker delta function*:

$$\delta(x,y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}$$

and the *zeta function*:

$$\zeta(x,y) = \begin{cases} 1 & \text{if } x \leq y \\ 0 & \text{if } x \not\leq y \end{cases}$$

So $\delta, \zeta \in \mathcal{F}$.

For functions $f, g \in \mathcal{F}$ the *convolution product* $f * g \in \mathcal{F}$ is:

$$\begin{cases} 0 & \text{if } x \not\leq y \\ \sum_{\{z : x \leq z \leq y\}} f(x,z)g(z,y) & \text{else} \end{cases}$$

Now let $\mathcal{F}' = \{f \in \mathcal{F} : \forall y \in X \; f(y,y) \neq 0\}$. This will be the *group of invertible functions* on $\mathbb{P}$.

For $f \in \mathcal{F}'$ define g as follows:

$$g(x,y) = \begin{cases} 0 & \text{if } x \neq y \\ \frac{1}{f(y,y)} & \text{if } x = y \\ -\frac{1}{f(y,y)} \sum_{\{z : x \leq z < y\}} g(x,z)f(z,y) & \text{else} \end{cases}$$

1. Let $\mathbb{P} = (\mathcal{P}([3]), \subseteq)$ and let $f = \zeta = \zeta_{\mathbb{P}}$. Now make a table with the sets in $\mathcal{P}([3])$ as the indices for the rows x and columns y with table entries, $g(x,y)$.

2. Now make a table of values for $f * g$.

3. And then make a table of values for $g * f$.

4. Let $\mathbb{D} = ([10], |)$ and let $f = \zeta = \zeta\mathbb{D}$. Make a table with the sets in $[10]$ as the indices for the rows x and columns y with table entries $g(x, y)$.

5. Now make a table of values for $f * g$.