

Last name _____

First name _____

LARSON—MATH 750—CLASSROOM WORKSHEET 03
Divisibility Poset

A *partial order* on a set X is a relation “ \leq ” on X that is reflexive, anti-symmetric and transitive. We call (X, \leq) a *partially ordered set* (or *poset*).

1. Let $X = \{2, 3, \dots, 10\}$ and define the divisibility relation “ $|$ ”: For $x, y \in X$, $x|y$ (that is, x divides y , or y is divisible by x) if there is an integer k such that $kx = y$.

Show that $(X, |)$ is a poset.

(a) Show “ $|$ ” is reflexive.

(b) Show “ $|$ ” is anti-symmetric.

(c) Show “ $|$ ” is transitive.

For $x, y, z \in X$, x covers y (or equivalently y is covered by x) if $y \leq x$ and $y \leq z \leq x$ implies that $z = x$ or $z = y$.

A Hasse diagram (or covering diagram) for a poset (X, \leq) is a representation of the elements of X together with a line between elements x and y if x covers y .

2. Draw the Hasse diagram for $(X, |)$.

3. What members of $(X, |)$ are incomparable?

4. Find all the maximal and minimal elements for $(X, |)$.

5. Does $(X, |)$ have a maximum (top) or minimum (bottom) element?

6. What could we add to $(X, |)$ to give it a minimum element? Explain.

7. What could we add to $(X, |)$ to give it a maximum element? Explain.

8. Is $(X, |)$ linearly ordered? Explain.

9. Find a linear extension of $(X, |)$.