Notation. We use $V = V(G)$ for the vertex set of a graph $G$ and $E = E(G)$ for the edge set. The order $n$ of the graph is the cardinality of $V$ and the size $m$ is the cardinality of the edge set.

Definition. An independent set in a graph is a set of vertices which are pair-wise non-adjacent. A maximum independent set (MIS) is a largest cardinality independent set. The independence number $\alpha$ is the cardinality of an MIS.

1. Let $G$ be the Petersen graph. Find a maximum independent set $I$ of $G$ and the independence number $\alpha = \alpha(G)$.

2. Find a spanning tree $T$ for the Petersen graph.

3. Find $\alpha(T)$.

4. Argue that, for any spanning subgraph $H$ of a graph $G$, that $\alpha(G) \leq \alpha(H)$. 

A subgraph of a graph $G = (V, E)$ is a graph $H = (V', E')$ where $V \subseteq V$ and $E' \subseteq E$. A subgraph $H$ is spanning if $V = V'$.

A partial order on a set $X$ is a relation “$\leq$" on $X$ that is reflexive, anti-symmetric and transitive. We call $(X, \leq)$ a partially ordered set (or poset).

We claim that, for any connected graph $G$, the set $X$ of connected spanning subgraphs of $G$ is a partial order on $G$ with the subgraph relation.

5. Check that the relation on $X$ is reflexive: that is, check that for any graph $G \in X$ that $G \leq G$. if

6. Check that the relation on $X$ is anti-symmetric: that is, check that for any graphs $G, G' \in X$ that $G \leq G'$ and $G' \leq G$ then $G = G'$.

7. Check that the relation on $X$ is transitive: that is, check that for any graphs $G, G', G'' \in X$ that $G \leq G'$ and $G' \leq G''$ implies that $G \leq G''$. 