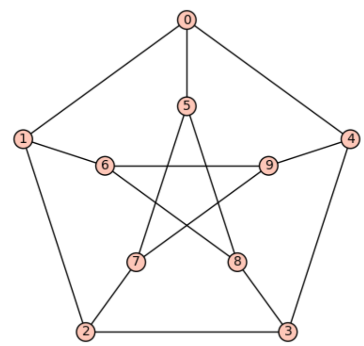
Last name ________________________

First name ________________________

LARSON—MATH 750—CLASSROOM WORKSHEET 02
Independence Number of a Graph

Notation. We use $V = V(G)$ for the vertex set of a graph G and $E = E(G)$ for the edge set. The *order* n of the graph is the cardinality of V and the *size* m is the cardinality of the edge set.

Definition. An *independent set* in a graph is a set of vertices which are pair-wise non-adjacent. A *maximum independent set* (MIS) is a largest cardinality independent set. The *independence number* α is the cardinality of an MIS.



1. Let G be the Petersen graph. Find a maximum independent set I of G and the independence number $\alpha = \alpha(G)$.

2. Find a spanning tree T for the Petersen graph.

3. Find $\alpha(T)$.

4. Argue that, for any spanning subgraph H of a graph G, that $\alpha(G) \leq \alpha(H)$.

A subgraph of a graph $G = (V, E)$ is a graph $H = (V', E')$ where $V \subseteq V$ and $E' \subseteq E$. A subgraph H is *spanning* if $V = V'$.

A *partial order* on a set X is a relation "$\leq$" on X that is reflexive, anti-symmetric and transitive. We call $(X, \leq)$ a *partially ordered set* (or *poset*).

We claim that, for any connected graph G, the set X of connected spanning subgraphs of G is a partial order on G with the subgraph relation.

5. Check that the relation on X is reflexive: that is, check that for any graph $G \in X$ that $G \leq G$. if

6. Check that the relation on X is anti-symmetric: that is, check that for any graphs $G, G' \in X$ that $G \leq G'$ and $G' \leq G$ then $G = G'$.

7. Check that the relation on X is transitive: that is, check that for any graphs $G, G', G'' \in X$ that $G \leq G'$ and $G' \leq G''$ implies that $G \leq G''$.