Last name _____

First name _____

LARSON—OPER 731—SAGE WORKSHEET 04
Linear Programming.

1. Log in to your Sage Cloud account.

   (a) Start Chrome browser.

   (b) Go to `http://cocalc.com`

   (c) Click "Sign In".

   (d) Click the project for our course.

   (e) Click "New", call it **s04**, then click "Sage Worksheet".

   Here's the linear program we'd like to solve:

   maximize: $x_1 + x_2 + x_3$

$$\text{subject to:} \quad \begin{array}{ccccccc} x_1 & + & x_2 & & & \leq & 1 \\ x_1 & & & + & x_3 & \leq & 1 \\ & & x_2 & + & x_3 & \leq & 1 \end{array}$$

   We will let `LP` be the name of our linear program. Of course, the name can be *anything*; it can be `Dantzig`, `D` or `Mathzilla`, anything that you are not already using for something else.

   We will also tell Sage that our objectiive is to find the **maximum** value of the objective function.

2. Evaluate `LP = MixedIntegerLinearProgram(maximization=True)`

   Now we will let x be the name of the variable vector, and also require that the vector entries be non-negative.

3. Evaluate `x = LP.new_variable(nonnegative=True)`.

   Now we will tell Sage what the objective function is. Notice that we are also implicitly giving the components of vector x the names $x[1]$, $x[2]$ and $x[3]$.

4. Evaluate `LP.set_objective(x[1] + x[2] + x[3])`.

   Now lets add our constraints.

5. Evaluate:

```
LP.add_constraint(x[1] + x[2], max = 1)
LP.add_constraint(x[1] + x[3], max = 1)
LP.add_constraint(x[2] + x[3], max = 1)
```

6. Now evaluate `LP.solve()` to solve the linear program. What do you get?

   This should print the maximum possible value of the objective function. (And there is probably a teeny error - all LP solvers are subject to some numerical instability.) All the work was done in the last step. If the LP is big this could take some time. Now let's see a feasible solution that attains the optimal value.

7. Evaluate `LP.get_values(x)`. What do you get?

   If you want only integer solutions for x—turning our problem into an Integer Programming problem. We can set x to be integer.

8. Evaluate `LP.set_integer(x)`. Then we can resolve. Evaluate `LP.solve()` and `LP.get_values(x)` again. What do you get? What does it mean?

   Now lets set up the dual of the last linear program. We found that the dual is:

   minimize: $y_1 + y_2 + y_3$

$$\text{subject to:} \quad \begin{array}{ccccccc} y_1 & + & y_2 & & & \geq & 1 \\ y_1 & & & + & y_3 & \geq & 1 \\ & & y_2 & + & y_3 & \geq & 1 \end{array}$$

   We'll call this system LPdual. Here are all the steps.

9. Evaluate:

```
LPdual = MixedIntegerLinearProgram(maximization=False)
y = LPdual.new_variable(nonnegative=True)
LPdual.set_objective(y[1] + y[2] + y[3])
LPdual.add_constraint(y[1] + y[2], min = 1)
LPdual.add_constraint(y[1] + y[3], min = 1)
LPdual.add_constraint(y[2] + y[3], min = 1)
LPdual.solve()
LPdual.get_values(y)
```

   What did you get? What does it mean?