LARSON—OPER 731—SAGE WORKSHEET 03

Matrices.

1. Log in to your Sage Cloud account.
   (a) Start Chrome browser.
   (b) Go to http://cloud.sagemath.com
   (c) Click “Sign In”.
   (d) Click class project.
   (e) Click “New”, call it s03, then click “Sage Worksheet”.

2. We can represent the system of linear equations
   \[
   \begin{align*}
   2x + y &= 5 \\
   x + 3y &= 7
   \end{align*}
   \]
   with the matrix
   \[
   A = \begin{bmatrix}
   2 & 1 & 5 \\
   1 & 3 & 7
   \end{bmatrix}
   \]
   Enter this in Sage by evaluating:
   \[A = \text{matrix}(2, 3, [2, 1, 5, 1, 3, 7])\]

3. Evaluate A to see your matrix.

4. Evaluate A.rref() to find a matrix that represents an equivalent system in row-reduced echelon form. What do you get?

5. Consider the system:
   \[
   \begin{align*}
   x + 3y &= 5 \\
   x + 3y &= 7
   \end{align*}
   \]
   Find a matrix that represents this system, and enter it in Sage. Then use Sage to find the row-reduced echelon form of this matrix. Then rewrite (on your own, without Sage) this as an equivalent system of linear equations and interpret.
6. Evaluate \( A=\text{matrix}(2,2,[1,2,3,4]) \), and \( b=\text{vector}([5,6]) \). Solve the matrix equation \( A\hat{x} = \hat{b} \) by evaluating \( A.\text{solve\_right}(b) \). What do you get?

7. Find the dot product of \( \hat{b} \) with itself \( (\hat{b} \cdot \hat{b}) \): evaluate \( b.\text{dot\_product}(b) \).

8. Find the length of \( \hat{b} \): evaluate \( b.\text{norm}() \). To get a numerical approximation: evaluate \( n(b.\text{norm}()) \).

9. Let \( x=A.\text{solve\_right}(b) \). Evaluate \( A*x \) to check your answer.

10. Let \( M=\text{matrix}(2,2,[1,2,3,4]) \). Evaluate \( M.\text{eigenvalues}() \) to find the eigenvalues of \( M \).

11. Evaluate \( M.\text{eigenvectors\_right}() \) to find the eigenvectors of \( M \). What does the output mean? Let \( x \) be one of the eigenvectors and \( \lambda \) be the corresponding eigenvalue. Check that \( M\hat{x} = \lambda\hat{x} \).

12. Is \( M \) invertible? Evaluate \( M.\text{is\_invertible}() \).

13. Evaluate \( \text{det}(M) \) to find the determinant of \( M \).

14. Evaluate \( M_{\text{inv}} = M.\text{inverse}() \) to find the inverse of \( M \). Check: evaluate \( M*M_{\text{inv}} \).

15. Evaluate \( M.\text{transpose}() \) to find the transpose of \( M \). Find the product of \( M \) and its transpose.

16. Let \( A \) be any \( 2 \times 3 \) matrix. Find its transpose. Find their product. What do you notice about the resulting matrix?