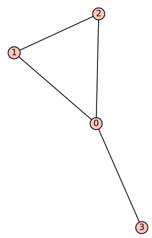
Last name ______________________

First name ______________________

LARSON—OPER 731—HOMEWORK WORKSHEET 05
Test 1 Review



Turn in a nicely written-up test review at test time.

Definitions State these clearly and precisely. Illustrate each concept with an example.

1. vertex packing
2. vertex cover.
3. matching
4. edge cover.
5. vertex-edge incidence matrix.
6. directed vertex-edge incidence matrix
7. convex hull of a point set X.
8. conical combination (of a set of vectors).
9. polytope.
10. extreme point (of a polytope).
11. facet (of a polytope).
12. dimension (of a polytope).
13. totally unimodular matrix.

    Theorems. State these clearly and precisely.

14. Duality Theorem.
15. Weyl's Theorem.
16. Farkas Lemma.
17. Minkowski's Theorem.
18. Total Unimodularity implies Integrality.
19. 4 Facts about Totally Unimodular Matrices.

Problems. Show your work, and explain.

20. Find the vertex-edge incidence matrix of the paw graph.

21. List all maximum vertex packings in the paw graph. Write characteristic vectors for each.

22. Write an Integer Program (IP) whose optimum is the size (cardinality) of a maximum vertex packing in the paw. Explain why this *models* the graph theoretic vertex packing (why is the optimum equal to the vertex packing number).

23. What is the *relaxation* (LP) corresponding to this IP?

24. Write this in the form $A\vec{x} \leq \vec{b}$.

25. Is A totally unimodular? Explain.

26. What are the advantages of the relaxation of the VPIP? Why might you relax?

27. What is the relationship between the optimal solution of the VPIP and its relaxation? Explain.

28. Find the dual of the VPLP. Explain.

29. Give a combinatorial interpretation of the dual (what does it tell you about the graph).

30. Find the optimal solution of VPLP and use the dual to prove that it is optimal.

31. Find the optimal solution of the VPIP and prove that it is optimal (using an argument that would work in any similar case—in particular, don't appeal to enumerating possible vertex sets—we can't do that in general).

32. Fourier-Motzkin Elimination. Use Fourier-Motzkin elimination to write an LP which is equivalent to the paw graph VPLP but which does not contain the variable x_0.

33. List all maximum matchings in the paw graph.

34. Write an IP that models the problem: find the cardinality of a maximum matching in the paw graph.

35. Let $X = \{\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\}$. Find a system of linear inequalities whose solutions are exactly $conv(X)$.

36. Let $\vec{v} = \begin{bmatrix}1\\2\end{bmatrix}$ and $\vec{w} = \begin{bmatrix}3\\1\end{bmatrix}$. Draw their conical combinations.

37. Then let $\vec{b} = \begin{bmatrix}3\\0\end{bmatrix}$. Find the vector $\vec{y}$ guaranteed by Farkas's Lemma.

38. Let $\mathcal{P}$ be the polytope formed by the intersection of half-spaces defined by the following three inequalities together with $x_i \geq 0$ (for $i = 1, 2, 3$). $\begin{array}{ccccccc} x_1 & + & x_2 & & & \leq & 1 \\ x_1 & + & & & x_3 & \leq & 1 \\ & & x_2 & + & x_3 & \leq & 1 \end{array}$

    Find the facets of $\mathcal{P}$.

39. **Prove** the vertex-edge incidence matrix of a bipartite graph is totally unimodular.