LARSON—OPER 731—HOMEWORK WORKSHEET 04
Barycentric Coordinates

Let $X^1$, $X^2$, and $X^3$ be non-collinear points in the plane. Let $X^*$ be a point in the triangle formed by these points. There are unique real numbers $\lambda_1$, $\lambda_2$, $\lambda_3$ in $[0, 1]$ where

$$X^* = \lambda_1 X^1 + \lambda_2 X^2 + \lambda_3 X^3.$$ 

The tuple $(\lambda_1, \lambda_2, \lambda_3)$ is called the barycentric coordinates of $X^*$.

1. Let $X^1 = (1, 1)$, $X^2 = (3, 2)$, and $X^3 = (2, 4)$. Find the barycentric coordinates of the point $X^* = (2, 3)$.

2. Prove that the barycentric coordinates of this point are unique.

3. Find a formula for the barycentric coordinates of an arbitrary point $X^*$ in the triangle formed by arbitrary non-colinear points $X^1$, $X^2$, and $X^3$ in the plane.