Matroid Polytopes

For a matroid $\mathcal{M}$ a polytope can be defined in terms of the characteristic vectors of its independent sets:

$$P_{I(\mathcal{M})} = \text{conv}\{\chi(S) : S \in I(\mathcal{M})\}$$

**Uniform Matroid.** Let $E(\mathcal{M}) = \{a, b, c, d, e\}$. Let $I(\mathcal{M})$ be the subsets of $E(\mathcal{M})$ with cardinality no more than 2.

1. Find $P_{I(\mathcal{M})}$ by finding $\{\chi(S) : S \in I(\mathcal{M})\}$.

**Linear Matroid.** Let $A = \begin{pmatrix} 0 & 1 & 2 & 4 \\ 1 & 0 & 3 & 5 \end{pmatrix}$.

Let $E(\mathcal{M})$ be the set consisting of the columns of $A$. Let $I(\mathcal{M})$ be the set consisting of the linearly independent subsets of $E(\mathcal{M})$.

2. Find $P_{I(\mathcal{M})}$ by finding $\{\chi(S) : S \in I(\mathcal{M})\}$. 
Graphical Matroid.

Let $G$ be the bow tie graph. $\mathcal{M}$ is the graphical matroid consisting of the forests of $G$. $E(\mathcal{M})$ is the edges of $G$. $I(\mathcal{M})$ is the forests of $G$.

3. Find $\mathcal{P}_{I(\mathcal{M})}$ by finding $\{\chi(S) : S \in I(\mathcal{M})\}$