Write up careful and complete answers.

**Concepts & Notation.** Give a careful definition and example for each concept.

1. What is a 1-factor? (And what is the difference from a perfect matching?)

2. Given a set $S \subseteq V(G)$, what is $o(G - S)$?

3. What is Tutte’s Condition?

4. Given a set $S \subseteq V(G)$, what is $def(S)$?

5. What is $def(G)$?

6. What is a factor-critical graph?

7. What is a near-perfect matching?

8. What is a feasible flow (Give a non-0-flow example)?

9. What is the value ($val(f)$) of a flow $f$?

10. What is a maximum flow?

11. What is a $f$-augmenting path?

12. What is the tolerance of an $f$-augmenting path $P$?

13. What is a source/sink cut $[S, T]$ in a network?

14. What is the capacity ($cap(S, T)$) of a source/sink cut $[S, T]$ in a network?
Theorems

15. What is Tutte’s Theorem?

16. What is the Berge-Tutte Formula?

17. What is Petersen’s Theorem?

18. What is the Max-Flow Min-Cut Theorem?

Proofs.

19. Prove: Any matching $M$ in a graph $G$ with order $n$ leaves at least $\text{def}(G)$ vertices unsaturated.

20. Explain the ideas of the proof of the Berge-Tutte Formula.

21. Prove the Weak Duality Theorem: If $f$ is a feasible flow and $[S,T]$ is a source/sink cut, then $\text{val}(f) \leq \text{cap}(S,T)$.

22. Prove: the Max-Flow Min-Cut Theorem

Algorithms

23. What is the main idea of Edmond’s Blossom Algorithm?

24. What is the Ford-Fulkerson Algorithm (Explain the steps)?
Problems. Explain as completely as you can.

25. Find the sets $D$, $A$, and $C$ from the Gallai-Edmonds Decomposition for the lantern graph.

26. Find a maximum deficiency set $T$ for the lantern graph. Is it maximal?

27. Find the auxiliary graph $H(T)$ for the lantern graph and your set $T$.

28. Show that $\text{def}(A) = \text{def}(G)$ for the lantern graph.

29. Find the sets $D$, $A$, and $C$ in the complete bipartite graph $K_{m,n}$ from the Gallai-Edmonds Decomposition.

30. Find a maximum deficiency set $T$ in $K_{m,n}$.

31. Show that $\text{def}(A) = \text{def}(G)$ for $K_{m,n}$.

32. Find the sets $D$, $A$, and $C$ in the cycle graph $C_n$ (with $n$ even) from the Gallai-Edmonds Decomposition.

33. Find the sets $D$, $A$, and $C$ in the cycle graph $C_n$ (with $n$ odd) from the Gallai-Edmonds Decomposition.

34. Give an example that shows that Petersen’s Theorem may not hold for a cubic graph with cut edges.
35. Find a maximum flow $f$ in this network (with indicated capacities) and argue that it is in fact maximum.

36. Find a minimum cut $[S, T]$ in this network.

37. How is the Max-Flow Min-Cut Theorem a Min-Max Relation and how does it provide a “certificate” for a maximum flow or a minimum cut?

38. How can network flow theory be used to find a maximum matching in a bipartite graph?