LARSON—MATH 656—Test 1 Review

Write up careful and complete answers.

Concepts & Notation

Give a careful definition and example for each concept.

1. What is a matching?
2. What does it mean for a matching to saturate a vertex?
3. What is the difference between a maximal and maximum matching?
4. If $M$ is a matching, what is an $M$-alternating path?
5. If $M$ is a matching, what is an $M$-augmenting path?
6. What is Hall’s Condition?
7. What is a vertex cover?
8. What is the notation for the vertex covering number—and the matching number?
9. What is a min-max relation?
10. What is an independent set?
11. What is the independence number?
12. What is an edge cover?
13. What is a dominating set?
14. What is the domination number $\gamma$?
15. What is the closed neighborhood $N[v]$ of a vertex $v$?
16. What is an independent dominating set?
17. What is a claw in a graph?
18. What is a claw-free graph?
19. What is a linear program?
20. What is a maximum weighted matching (of a weighted graph)?
21. What is an example of an application of finding a maximum weighted matching in a bipartite graph?
22. What is a transversal?
23. What is a cover $(u, v)$ of a weighted graph?
24. What is the cost \( c(u, v) \) of a weighted graph?

25. What is the dual problem of finding a weighted bipartite matching in a weighted graph?

26. Given \( n \) “men”, \( n \) “women” and linearly ordered preferences for each, what is an unstable pair?

27. Given \( n \) “men”, \( n \) “women” and linearly ordered preferences for each, what is an stable matching?

28. What is a 1-factor? (And what is the difference from a perfect matching?)

29. Given a set \( S \subseteq V(G) \), what is \( o(G - S) \)?

30. What is Tutte’s Condition?

**Theorems**

31. What is König’s Theorem (also called the König-Egerváry Theorem)?

32. What is Hall’s Theorem?

33. What is the Marriage Theorem?

34. What are the Gallai Identities?

35. What is Berge’s Theorem?

36. What is the Symmetric Difference Lemma?

37. What is the Duality Property for maximum weighted matchings in bipartite graphs?

38. What is Tutte’s Theorem?

**Proofs**

Give a careful proof of each of the following theorems.

39. Use Hall’s Theorem to prove König’s Theorem.


41. Prove: If a graph is claw-free then it has an independent set of size \( \gamma \).

42. Prove: A set of vertices is an independent dominating set if and only if it is a maximal dominating set.

**Algorithms**

43. What is the the **Augmenting Path Algorithm** (What is an algorithm for finding a maximum matching and minimum vertex cover in a bipartite graph?)
44. What is the Hungarian Method? (Make sure you understand it. There will definitely be a Hungarian Method application on the test).

45. Given \( n \) “men”, \( n \) “women” and linearly ordered preferences for each, what is an algorithm for producing a stable matching?

**Problems**

Explain as completely as you can.

46. What is the relationship between independent sets and vertex covers?

47. What can we say about \( k \)-regular bipartite graphs?

48. Why can we always assume our graph is \( K_{n,n} \) for the problem of finding a maximum weighted matching in a bipartite graph?

49. What is the *dual* problem of finding a weighted bipartite matching in a weighted graph?

50. Why is the problem of finding the maximum sum of a transversal equivalent to the problem of finding a maximum weight matching in a bipartite graph?

51. How is the Duality Property a Min-Max Relation and how does it provide a “certificate” for a maximum weighted matching or a minimum weighted cover?