Organizational Notes

1. Don’t forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like “Math 656 c25 notes”).
2. The VCU Discrete Math Seminar is every Wednesday.
3. Read ahead! We’re talking about Network Flow problems (Sec. 4.3)

Review

1. What is a network?

2. What is a flow? What is $f^+(v)$ and $f^−(v)$?

3. What is a feasible flow? What are capacity constraints?

4. What are conservation constraints?

5. What is the value $\text{val}(f)$ of a flow $f$? What is a maximum flow?

6. What is the tolerance of an edge $e$, of a path $P$? What is a $f$-augmenting path?

7. (Lemma) If $P$ is an $f$-augmenting path with tolerance $z$ then changing flow by $+z$ on edges followed forward by $P$ and by $-z$ on edges followed backward by $P$ produces a feasible flow $f'$ with $\text{val}(f') = \text{val}(f) + z$.

8. What is a source/sink cut $[S,T]$? What is the capacity, $\text{cap}(S,T)$, of a cut $[S,T]$?

9. If $U \subseteq V(G)$, what is $f^+(U)$ and $f^−(U)$?

10. (Lemma) If $U$ is a set of nodes in a network, then the net flow out of $U$ is the sum of the net flows out of the nodes of $U$, that is,

$$f^+(U) - f^−(U) = \sum_{v \in U} [f^+(v) - f^−(v)].$$
Notes

1. **(Lemma)** If $[S, T]$ is a source/sink cut, then the net flow of a flow $f$ out of $S$ equals the net flow into $T$. Furthermore, the net flow out of any source/sink cut is constant (and also equals the net flow out of $s$, and also equals $\text{val}(f)$).

2. **(Weak Duality)** If $f$ is a feasible flow and $[S,T]$ is a source/sink cut, then $\text{val}(f) \leq \text{cap}(S, T)$.

3. What is the **minimum cut** problem?

4. What is the **Ford-Fulkerson labeling algorithm**?

5. **(Max-flow Min-cut Theorem—AKA Ford-Fulkerson Theorem)** In every network, the maximum value of a feasible flow equals the minimum capacity of a source/sink cut.