Organizational Notes

1. Don’t forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like “Math 656 c22 notes”).

2. The VCU Discrete Math Seminar is every Wednesday.

3. h07 (the Gallai-Edmonds decomposition worksheet) is due today.

4. Read ahead! Next up we’ll talk about Network Flow problems (Sec. 4.3)

Concepts & Notation

• Edmonds-Gallai Decomposition (West paper).

• Petersen’s Theorem (Sec. 3.3).

• Network Flows (Sec. 4.3).

Review

1. (Gallai-Edmonds Structure Theorem). Let $A$, $C$, $D$, be the sets in the Gallai-Edmonds Decomposition of a graph $G$. Let $G_1, \ldots, G_k$ be the components of $G[D]$. If $M$ is a maximum matching in $G$ then:

   (a) $M$ covers $C$ and matches $A$ into distinct components of $G[D]$.

   (b) Each $G_i$ is factor-critical and $M$ restricts to a near-perfect matching on $G_i$.

   (c) If $S \subseteq A$ is non-empty then $N_G(S)$ has a vertex in at least $|S| + 1$ of $G_1, \ldots, G_k$.

   (d) $def(A) = def(G) = k - |A|$. 
Notes

1. (Petersen’s Theorem) If a graph has a perfect matching and no cut edges then it has a perfect matching.

2. What is a directed graph?

3. What is a network?

4. What is the capacity $c(e)$ of an edge $e$?

5. What are source and sink vertices?

6. What is a flow? What is $f^+(v)$ and $f^-(v)$?

7. What is a feasible flow? What are capacity constraints?

8. What are conservation constraints?

9. What is the value $val(f)$ of a flow $f$?

10. What is a maximum flow?

11. What is a $f$-augmenting path?

12. What is the tolerance of a flow?

13. (Lemma) If $P$ is an $f$-augmenting path with tolerance $z$ then changing flow by $+z$ on edges followed forward by $P$ and by $-z$ on edges followed backward by $P$ produces a feasible flow $f'$ with $vaf(f') = val(f) + z$. 