Organizational Notes

1. Don’t forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like “Math 656 c18 notes”).

2. The VCU Discrete Math Seminar is every Wednesday.

3. Read ahead! Next up we’ll talk the Gallai-Edmonds Matching Decomposition (as described in the West paper).

Concepts & Notation

- Sec. 3.3: general (cardinality) matching, Tutte’s Theorem, Edmonds-Gallai Decomposition.

Review

1. What is Tutte’s Theorem?

2. What is the Berge-Tutte Formula?

3. **Claim** Any matching leaves at least $\text{def}(G)$ vertices unsaturated.

4. **Parity Lemma**: $o(G - S) - |S| \equiv n \pmod{2}$.

5. **Auxiliary Graph** $H(T)$. If $T$ is a maximal maximum deficiency set, define the graph $H(T)$ with vertex set $Y$ consisting of one vertex for each (odd) component of $G - T$, the vertices $T$ and $y \in Y$ adjacent to $v \in T$ if any vertex in the component corresponding to $y$ is adjacent to $v$. ($H(T)$ is a $T - Y$-bighraph).
Notes

1. **Maximal Maximum Deficiency Set Lemma** Let $T$ be a maximal maximum deficiency set. Let $u$ be a vertex of an odd component $C$ of $G - T$. Then (1) $C - u$ satisfies Tutte’s condition, and (2) the components of $G - u$ are all odd.

2. (Lemma). $H(T)$ has a matching that covers $T$.

3. (Theorem) (Berge-Tutte Formula) $\nu = \frac{1}{2}(n - \text{def}(G))$.

4. (Theorem) (Tutte’s Theorem) A graph $G$ has a perfect matching if and only if for every $S \subseteq V(G)$ $\sigma(G - S) \leq |S|$.

5. A vertex $v$ in a graph is either (1) covered by every maximum matching (set $B$), or (2) not covered by every maximum matching (set $D$). A vertex is either (1) has a neighbor outside $B$ (set $D$) or (2) does not (set $C$). The **Gallai-Edmonds Decomposition** is the partition of $V(G)$ into sets $C$, $A$ and $D$. 