Organizational Notes

1. Don’t forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like “Math 656 c17 notes”).

2. The VCU Discrete Math Seminar is every Wednesday.

3. Read ahead! Next up we’ll talk the Gallai-Edmonds Matching Decomposition (as described in the West paper).

Concepts & Notation

- Sec. 3.3: general (cardinality) matching, Tutte’s Theorem, Edmonds-Gallai Decomposition.

Review

1. What is Tutte’s Theorem?

2. (Notation) What is $\text{def}(S)$?

3. (Notation) What is $\text{def}(G)$?

4. What is the Berge-Tutte Formula?

5. Claim Any matching leaves at least $\text{def}(G)$ vertices unsaturated.
Notes

1. **Parity Lemma**: \( o(G - S) - |S| \equiv n \pmod{2} \).

2. **Maximal Maximum Deficiency Set Lemma** Let \( T \) be a maximal maximum deficiency set. Let \( u \) be a vertex of an odd component \( C \) of \( G - T \). Then (1) \( C - u \) satisfies Tutte’s condition, and (2) the components of \( G - u \) are all odd.

3. **Auxilliary Graph \( H(T) \)**. If \( T \) is a maximal maximum deficiency set, define the graph \( H(T) \) with vertex set \( Y \) consisting of one vertex for each (odd) component of \( G - T \), the vertices \( T \) and \( y \in Y \) adjacent to \( v \in T \) if any vertex in the component corresponding to \( y \) is adjacent to \( v \). (\( H(T) \) is a \( T - Y \)-bigraph).

4. (Lemma). \( H(T) \) has a matching that covers \( T \).

5. Theorem (Berge-Tutte Formula) \( \nu = \frac{1}{2}(n - def(G)) \).