Organizational Notes

1. Don’t forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like “Math 656 c12 notes”).
2. The VCU Discrete Math Seminar is every Wednesday.
3. Homework #3 (h03) is due next Monday.
4. Read ahead! Next up we’ll talk about bipartite, weighted, stable and general matching algorithms (Sec. 3.2, Sec. 3.3).

Concepts & Notation

• Sec. 3.2: maximum bipartite matching algorithm, maximum weighted bipartite matching algorithm, transversal, Assignment Problem, stable matching.
• Sec. 3.3: general (cardinality) matching, Tutte’s Theorem, Edmonds-Gallai Decomposition.

Review

1. Why is the problem of finding the maximum sum of a transversal equivalent to the problem of finding a maximum weight matching in a bipartite graph?

2. (Notation). What is a cover \((u, v)\) and cost \(c(u, v)\)?

3. What is the dual problem of finding a weighted bipartite matching in a weighted graph?

4. (Duality Property): For a perfect matching \(M\) and cover \((u, v)\) in a weighted bipartite graph \((1)\) \(c(u, v) \geq w(M)\).

5. (Duality Property): For a perfect matching \(M\) and cover \((u, v)\) in a weighted bipartite graph \((2)\) \(c(u, v) = w(M)\) if and only if for every edge \(x_i y_j \in M\) \(u_i + v_j = w_{i,j}\).

6. How is the Duality Property a Min-Max Relation and how does it provide a “certificate” for a maximum weighted matching or a minimum weighted cover?
Notes

1. (Review): What does the augmenting path algorithm produce for a bipartite graph?

2. Given a weighted $X - Y$-bigraph (which we can assume to be $K_{n,n}$) with non-negative weights $\{w_{i,j}\}$ and (not necessarily optimal) cover $(u, v)$, what is the excess of an edge $x_iy_j$?

3. What is the Hungarian Method?

4. Why is the Hungarian method guaranteed to terminate?

5. Why is the Hungarian method guaranteed to produce a maximum weighted matching?

6. Given $n$ “men”, $n$ “women” and linearly ordered preferences for each, what is an unstable pair?

7. Given $n$ “men”, $n$ “women” and linearly ordered preferences for each, what is an stable matching?

8. Given $n$ “men”, $n$ “women” and linearly ordered preferences for each, what is an algorithm for producing a stable matching?

9. Is this algorithm biased (does it produce similar outputs regardless of whether men’s or women’s preferences are favored)?

10. Why is this algorithm guaranteed to terminate?

11. Why is this algorithm guaranteed to produce a stable matching?