Last name _____

First name _____

LARSON—OPER 635—SAGE WORKSHEET 05
More Linear Programming.

1. Log in to your Sage Cloud account.

   (a) Start Firefox or Chrome browser.

   (b) Go to `http://cloud.sagemath.com`

   (c) Click "Sign In".

   (d) Click project **OPER 635**.

   (e) Click "New", call it **s05**, then click "Sage Worksheet".

   Here's the linear program we'd like to solve:

   maximize: $5x_1 + 4x_2 + 3x_3$

$$\text{subject to:}\quad \begin{array}{ccccccc} 2x_1 & + & 3x_2 & + & x_3 & \leq & 5 \\ 4x_1 & + & x_2 & + & 2x_3 & \leq & 11 \\ 3x_1 & + & 4x_2 & + & 2x_3 & \leq & 8 \end{array}$$

   We will let `LP` be the name of our linear program. We will tell Sage that our objective is to find the **maximum** value of the objective function. Then we let x be the name of the variable vector, and also require that the vector entries be non-negative. We then tell Sage what the objective function is. Notice that we are also implicitly giving the components of vector x the names $x[1]$, $x[2]$ and $x[3]$. Finally we add our constraints, solve, and ask for a feasible solution that realizes this optimum value.

2. Evaluate:

```
LP = MixedIntegerLinearProgram(maximization=True)
x = LP.new_variable(nonnegative=True)
LP.set_objective(5*x[1] + 4*x[2] + 3*x[3])
LP.add_constraint(4*x[1] + x[2]+ 2*x[3], max = 11)
LP.add_constraint(2*x[1] + 3*x[2] + x[3], max = 5)
LP.add_constraint(3*x[1] + 4*x[2] + 2*x[3], max = 8)
LP.solve()
LP.get_values(x)
```

3. What did you get? What does it mean?

LP is written in the *standard form*:

$$\max \bar{c}^T \bar{x}$$

$$A\bar{x} \leq \bar{b}$$

$$x_i \geq 0$$

4. What are $\bar{c}$, $\bar{b}$ and A (that is, find them)?

The *dual* of this system is the system:

$$\min \bar{b}^T \bar{y}$$

$$A^T \bar{y} \geq \bar{c}$$

$$y_j \geq 0$$

5. Write out the objective and constraints for the dual system in our example.

6. Evaluate:

```
LPdual = MixedIntegerLinearProgram(maximization=False)
y = LPdual.new_variable(nonnegative=True)
LPdual.set_objective(11*y[1] + 5*y[2] + 8*y[3])
LPdual.add_constraint(4*y[1] + 2*y[2]+ 3*y[3], min = 5)
LPdual.add_constraint(y[1] + 3*y[2] + 4*y[3], min = 4)
LPdual.add_constraint(2*y[1] + y[2] + 2*y[3], min = 3)
LPdual.solve()
LPdual.get_values(y)
```

7. What did you get? What does it mean? Does it agree with the Duality Theorem?