Last name _____

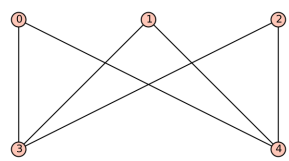
First name _____

LARSON—OPER 635—CLASSROOM WORKSHEET 17
König-Egerváry Theorem

A *matching* in a graph is a set of vertex disjoint arcs. A *maximum matching* in a graph is a maximum cardinality matching. The *matching number* ν is the cardinality of a maximum matching.

A *(node) cover* in a graph is a set of nodes which are incident to all the arcs in the graph. A *minimum cover* in a graph is a minimum cardinality cover. The *covering number* τ is the cardinality of a minimum cover.

The *König-Egerváry Theorem* says that, for a bipartite graph, $\nu = \tau$.



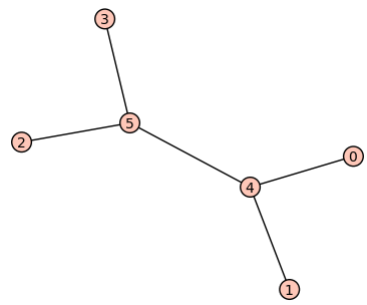
1. $k_{2,3}$ is bipartite. Partition the vertices into 2 independent sets. Add a source s to all vertices in one set X, a sink t to all vertices in the other set Y. Direct all arcs in the direction from the source to the sink and make all capacities 1.

2. Find a maximum flow $f = \{x_{i,j}\}$. Let M be the set of arcs from X to Y with positive flow. Check that $\nu \geq |M|$.

3. Find the residual network.

4. Let S be the set of nodes reachable from the source. Find S and the cut $[S, \bar{S}]$. Check that $val(f) = cap([S, \bar{S}])$.

5. Let C be the nodes $\{x : (s, x) \in [S, \bar{S}]\} \cup \{y : (y, t) \in [S, \bar{S}]\}$. Check that C is a cover of $k_{2,3}$ and also that $cap([S, \bar{S}]) \geq |C|$.
   Note that $|C| \geq \tau$.



6. Find a maximum matching in the *killer* and find ν.

7. Find a minimum cover for the killer and find τ.

8. Check that the killer is bipartite, and find a bipartition (X, Y) of the node set.

9. Add a source s to all vertices in one set X, a sink t to all vertices in the other set Y. Direct all arcs in the direction from the source to the sink and make all capacities 1.

10. Find a maximum flow $f = \{x_{i,j}\}$. Let M be the set of arcs from X to Y with positive flow. Check that $\nu \geq |M|$.

11. Find the residual network.

12. Let S be the set of nodes reachable from the source. Find S and the cut $[S, \bar{S}]$. Check that $val(f) = cap([S, \bar{S}])$.

13. Let C be the nodes $\{x : (s, x) \in [S, \bar{S}]\} \cup \{y : (y, t) \in [S, \bar{S}]\}$. Check that C is a cover of the killer and also that $cap([S, \bar{S}]) \geq |C|$. Note that $|C| \geq \tau$.