Last name _____

First name _____

LARSON—MATH 591—Test 1 Review

You should know the following definitions, theorems, examples, and proofs for the test. Write out careful definitions, theorem statements, proofs, and solutions.

Definitions & Notation

1. $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{C}$, $\mathbb{F}_2$.
2. What are the main properties of a field? Explain why $\mathbb{Z}$ is not a field.
3. **algebraically closed field**.
4. **monomial** in $x_1, \ldots, x_n$.
5. **total degree** of a monomial.
6. **polynomial** in $x_1, \ldots, x_n$.
7. **zero polynomial**.
8. k^n, for a field k.
9. Explain how any polynomial f in variables $x_1, \ldots, x_n$ defines a function from k^n to k.
10. What is a zero function? What does it mean for a polynomial f to define a zero function?
11. $k[x_1, \ldots, x_n]$.
12. What properties does $k[x_1, \ldots, x_n]$ have? Is it a field?
13. $k(x_1, \ldots, x_n)$.
14. Affine **variety** $\mathbb{V}(f_1, \ldots, f_n)$.
15. **linear variety**.
16. **twisted cubic**.
17. **ideal** in $k[x_1, \ldots, x_n]$.
18. $\langle f_1, \ldots, f_s \rangle$ for $f_1, \ldots, f_s$ in $k[x_1, \ldots, x_n]$.

Theorems

19. Relationship between zero polynomials and zero functions.
20. The union and intersection of varieties theorem (with the formulas for these).
21. Equality of ideals gives equality of varieties.

Examples & Problems

22. Give some examples of both finite and infinite fields.

23. Find a monomial in x, y, z with total degree 10.

24. Give an example of a non-zero polynomial which defines a zero function.

25. Give an explicit description of $\mathbb{R}[x, y]$.

26. Show how the polynomial $x^2y^3z^5 + yz + xz^2 + xy + xyz + y^2z^3$ in $\mathbb{R}[x, y, z]$ can be written in the form $\sum g_i(y, z)x^i$. Find the g_i's.

27. Draw $\mathbb{V}(xy, xz)$. Explain how you got it.

28. Write the defining equations for $\mathbb{V}(y - x - 3)$.

29. Draw $\mathbb{V}(y - x - 3)$.

30. Parameterize the points on $\mathbb{V}(y - x - 3)$.

31. Show that $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ gives a parameterization of the unit circle $x^2 + y^2 = 1$.

32. Give an example of a finite variety.

33. Give an example of an empty variety.

34. Give an example of a variety with 2 different sets of defining polynomials.

35. Define $\langle x^2 - 4, y^2 - 1\rangle$.

36. Define $\langle x + y, x - y\rangle$ in $\mathbb{R}[x, y]$.

37. Show x^2 in $\langle x + y, x - y\rangle$.

Proofs

38. Show $\langle x, y\rangle = \langle x + y, x - y\rangle$.

39. Show $\langle f_1, \ldots, f_s\rangle$ is an ideal in $k[x_1, \ldots, x_n]$.