

Last name _____

First name _____

LARSON—MATH 591—SAGE WORKSHEET 08
Bonus.

This worksheet will count as a **Bonus** Homework assignment.

1. Log in to your Sage Cloud account.
 - (a) Start Firefox or Chrome browser.
 - (b) Go to <http://cloud.sagemath.com>
 - (c) Click “Sign In”.
 - (d) Click project **Math 591**.
 - (e) Click “New”, call it **s08**, then click “Sage Worksheet”.

First we’ll define the polynomial ring R with variables x, y, z and *real number* coefficients. In the following command “R” is the user-chosen name for the ring, “x,y,z” are the user-chosen variable names, “RR” says the coefficients are real numbers. All user-chosen elements can be changed. We give the terms *lex* order. This too is a user-chosen option; other monomial orders are possible.

2. Evaluate:

```
R.<x,y,z> = PolynomialRing(RR, 3, order='lex')
```

Consider the following system of equations:

$$x^2 + y^2 + z^2 = 1,$$

$$x^2 + z^2 = y,$$

$$x = z.$$

3. We’d like to investigate the variety $\mathbb{V}(f_1, f_2, f_3)$ that consists of the solutions of these equations. Define the f ’s. For Sage, call them $f1, f2$ and $f3$.
4. Now find the ideal they generate: `I=ideal(f1,f2,f3)`.
5. Since we did this example in Homework #12, it might interest you to find the Groebner basis of this ideal. Evaluate: `I.groebner_basis()`.
6. If the variety is 0-dimensional (that is, it consists of discrete points), Sage can find those points. Evaluate: `I.variety()`

7. It is crucial that we used the real field \mathbb{R} in our definition of the polynomial ring. Evaluate:

```
R2.<x,y,z> = PolynomialRing(QQ, 3, order='lex')
```

8. Redefine I (so that now I is an ideal in R_2) and evaluate $I.\text{variety}()$ again. Can you explain the output?

For the remaining problems, you can solve them all using Sage commands that we've seen on this and previous worksheets. In your answer, write the commands you used *and* Sage's output.

9. Chp. 1, Sec. 5: #8, 9.
10. Chp. 2, Sec. 8: #1,2,3,4. (Use " \mathbb{C} " for complex numbers in your polynomial ring definition, as needed).