Last name ______________________

First name ______________________

LARSON—MATH 591—SAGE WORKSHEET 07
Polynomials in $\mathbb{R}[x, y, z]$.

1. Log in to your Sage Cloud account.
   (a) Start Firefox or Chrome browser.
   (b) Go to `http://cloud.sagemath.com`
   (c) Click "Sign In".
   (d) Click project **Math 591**.
   (e) Click "New", call it **s07**, then click "Sage Worksheet".

   First we'll define the polynomial ring R with variables x, y, z and rational coefficients. In the following command "R" is the user-chosen name for the ring, "x,y,z" are the user-chosen variable names, "QQ" says the coefficients are real numbers. All user-chosen elements can be changed. We give the terms *lex* order. This too is a user-chosen option; other monomial orders are possible.

2. Evaluate:

```
R.<x,y,z> = PolynomialRing(QQ, 3, order='lex')
```

3. Now evaluate `R` to see what Sage thinks R is.

4. Evaluate: `x > y`.

5. Evaluate: `z > x`.

6. Evaluate: `x**2 > x`.

7. Evaluate: `y**2 > x`.

8. Evaluate: `x * y**2 > y**3 * z**4`.

We can define an ideal I in this polynomial ring generated by some number of polynomials using the *ideal* method.

9. For instance, evaluate: `I=ideal(x**2 - z - 1, z**2 - y - 1, x*y**2 - x - 1)`.

10. Now evaluate `I` to see what Sage thinks I is.

11. You can check that the generators are in this ideal. Evaluate: `x**2 - z - 1 in I`.

12. Is this ideal the whole ring? It is enough to test if any constant is in I.
    Evaluate: `1 in I`.

13. To find a *Groebner basis* for I we can use the *groebner_basis* method. Evaluate: `I.groebner_basis()`.

The Groebner basis is defined in terms of the *leading terms* of the ideal and, thus, depends on the chosen monomial order. Lets re-do everything for the *graded lex* (or *grlex* or *deglex*) order. We'll call this ring $R2$.

14. Evaluate:

```
R2.<x,y,z> = PolynomialRing(QQ, 3, order='deglex')
```

15. Evaluate: `x * y**2 > y**3 * z**4`.

16. Let $I2$ be the ideal generated by the same polynomials as before—but in a ring where the order has changed.
    Evaluate: `I2=ideal(x**2 - z - 1, z**2 - y - 1, x*y**2 - x - 1)`.

17. Now evaluate: `I2.groebner_basis()`.