**Last name** ______________________

**First name** ______________________

# LARSON—MATH 591—SAGE WORKSHEET 06
## GCD for Polynomials in $\mathbb{R}[x]$.

1. Log in to your Sage Cloud account.
   (a) Start Firefox or Chrome browser.
   (b) Go to `http://cloud.sagemath.com`
   (c) Click "Sign In".
   (d) Click project **Math 591**.
   (e) Click "New", call it **s06**, then click "Sage Worksheet".

First we'll define the polynomial ring $R$ with variable $x$ and real coefficients. In the following command "R" is the user-chosen name for the ring, "x" is the user-chosen variable names, "RR" says the coefficients are real numbers. All user-chosen elements can be changed.

2. Evaluate:

```
R.<x> = RR['x']
```

3. Now evaluate `R` to see what Sage thinks $R$ is.

Now that we've defined $x$ as the polynomial ring variable, we can define polynomials in $x$ as well as ideals of polynomials. Let's define two polynomials $f$, $g$ in $x$.

4. Evaluate:

```
f=x^2
g=x+1
```

5. Now evaluate `f` to see what Sage thinks $f$ is, and evaluate `type(f)` to see what *kind* of object Sage thinks $f$ is.

6. To test is $f$ is in the ring $R$, evaluate `f in R`. What do you get?

7. To find the GCD of $f$ and $g$ in the polynomial ring $R$, evaluate `gcd(f,g)`.

One thing we've proved is that the ideal generated by $f$ and $g$ equals the ideal generated by their GCD. In symbols, this says $\langle f, g\rangle = \langle GCD(f, g)\rangle$.

8. Let's check. Evaluate `ideal(f,g)==ideal(gcd(f,g))`. (Recall that `ideal(f,g)` is Sage's code for $\langle f, g\rangle$.)

9. Use Sage to find a single generator for the ideal $\langle x^4 - 1, x^6 - 1\rangle$.

You can find the GCD of 3 or more polynomials iteratively. You can (and should) prove that GCD(f,g,h)=GCD(GCD(f,g),h).

10. Use this idea to find GCD($x$,$x^2$,$x^3$). Of course the answer is $x$. But use the above idea to check that Sage gives the same result.

11. Now find GCD($x^4 + x^2 + 1$, $x^4 - x^2 - 2 * x$, $x^3 - 1$).

The *Ideal Membership Problem* is: given an ideal $I$ and a polynomial $p$, is $p \in I$? Assume $I = \langle f, g\rangle$, and $h = GCD(f, g)$. We proved $\langle f, g\rangle = \langle h\rangle$. To test whether $p \in \langle h\rangle$ is *easy*. $p \in \langle h\rangle$ *means* that $h|p$. That's it.

12. Let's use Sage to determine if $x^2 \in \langle x+1, x-1\rangle$ There are now a few ways we can do this. One way is to by checking this last property. Evaluate: `gcd(x+1,x-1).divides(x**2)`.

Now use Sage to find whether $x^2 - 4$ is in $\langle x^3 + x^2 - 4 * x - 4, x^3 - x^2 - 4 * x + 4, x^3 - 2 * x^2 - x + 2\rangle$ (or, in other words, whether $x^2$ can be written as a polynomial combination of $x^3 + x^2 - 4 * x - 4$, $x^3 - x^2 - 4 * x + 4$, and $x^3 - 2 * x^2 - x + 2$).

13. What command(s) would you write (there are a variety of things you can do that will work)?

14. Is it in the ideal?