

Last name _____

First name _____

LARSON—MATH 591—SAGE WORKSHEET 04
Varieties.

1. Log in to your Sage Cloud account.
 - (a) Start Firefox or Chrome browser.
 - (b) Go to <http://cloud.sagemath.com>
 - (c) Click “Sign In”.
 - (d) Click project **Math 591**.
 - (e) Click “New”, call it **s04**, then click “Sage Worksheet”.

Here we will look at examples of drawing varieties and finding descriptions of varieties.

2. First we'll tell Sage we plan to use y and z as variables. (Unless you're defined x as something else Sage always assumes x is a variable.) Evaluate:

```
var("y")
var("z")
```

3. Let's draw the variety $\mathbb{V}(xy - x^3 + 1) \subset \mathbb{R}^2$. Evaluate:

```
implicit_plot(x*y-x^3+1==0, (x,-10,10),(y,-10,10))
```

4. If only an expression is given `implicit_plot` assumes that you mean it to be equal to 0. So you could just do:

```
implicit_plot(x*y-x^3+1, (x,-10,10),(y,-10,10))
```

5. What command would you type to get a sketch of $\mathbb{V}(x^2 + 4y^2 + 2x - 16y + 1)$.

6. If you want to sketch $\mathbb{V}(z^2 - x^2 - y^2) \subseteq \mathbb{R}^3$, you have 3 variables and you need `implicit_plot3d`. Evaluate:

```
implicit_plot3d(z^2-x^2-y^2,(x,-10,10),(y,-10,10),(z,-10,10))
```

If you want to sketch a variety with more than one defining polynomial we proved that you can find the graph these separately and find their intersection.

7. Find $\mathbb{V}(x + y, x - y)$ by graphing. Evaluate:

```
implicit_plot(x+y,(x,-10,10),(y,-10,10))+  
implicit_plot(x-y,(x,-10,10),(y,-10,10),color="red")
```

8. To find a description of $\mathbb{V}(x + y, x - y)$, you can try `solve`. Evaluate:

```
solve([x+y,x-y],x,y)
```

Sage can parameterize the solutions to some varieties.

9. First let's visualize $\mathbb{V}(x + y - 3, 2x + 2y - 6)$. Evaluate:

```
implicit_plot(x+y-3,(x,-10,10),(y,-10,10))+  
implicit_plot(2*x+2*y-6,(x,-10,10),(y,-10,10), color="red")
```

(What happened? Where's the other graph???)

10. Find a parametrization *by hand* of $\mathbb{V}(x + y - 3, 2x + 2y - 6)$.

11. Now try `solve`. Evaluate:

```
solve([x+y - 3, 2*x+2*y - 6],x,y)
```

What is the meaning of the answer Sage gave you?

12. What command would you type to get a nice picture of the twisted cubic $\mathbb{V}(y - x^2, z - x^3)$?

13. Find a description of the points on the twisted cubic. (What is the meaning of the answer Sage gives you?) Evaluate:

```
solve([y-x^2,z-x^3],x,y,z)
```

14. What would you type to find a *description* of $\mathbb{V}(x^4 - zx, x^3 - yx)$?