1. Log in to your Sage Cloud account.
   
   (a) Start Firefox or Chrome browser.
   
   (b) Go to http://cloud.sagemath.com
   
   (c) Click “Sign In”.
   
   (d) Click project Math 591.
   
   (e) Click “New”, call it s04, then click “Sage Worksheet”.

Here we will look at examples of drawing varieties and finding descriptions of varieties.

2. First we’ll tell Sage we plan to use $y$ and $z$ as variables. (Unless you’re defined $x$ as something else Sage always assumes $x$ is a variable.) Evaluate:

   \[
   \text{var("y")}
   \]
   \[
   \text{var("z")}
   \]

3. Let’s draw the variety $V(xy - x^3 + 1) \subset \mathbb{R}^2$. Evaluate:

   \[
   \text{implicit_plot}(x*y-x^3+1==0, (x,-10,10),(y,-10,10))
   \]

4. If only an expression is given \texttt{implicit_plot} assumes that you mean it to be equal to 0. So you could just do:

   \[
   \text{implicit_plot}(x*y-x^3+1, (x,-10,10),(y,-10,10))
   \]

5. What command would you type to get a sketch of $V(x^2 + 4y^2 + 2x - 16y + 1)$.

6. If you want to sketch $V(z^2 - x^2 - y^2) \subset \mathbb{R}^3$, you have 3 variables and you need \texttt{implicit_plot3d}. Evaluate:

   \[
   \text{implicit_plot3d}(z^2-x^2-y^2, (x,-10,10),(y,-10,10),(z,-10,10))
   \]

If you want to sketch a variety with more than one defining polynomial we proved that you can find the graph these separately and find their intersection.
7. Find $\mathcal{V}(x + y, x - y)$ by graphing. Evaluate:

```python
implicit_plot(x+y,(x,-10,10),(y,-10,10)) +
implicit_plot(x-y,(x,-10,10),(y,-10,10),color="red")
```

8. To find a description of $\mathcal{V}(x + y, x - y)$, you can try `solve`. Evaluate:

```python
solve([x+y,x-y],x,y)
```

Sage can parameterize the solutions to some varieties.

9. First let's visualize $\mathcal{V}(x + y - 3, 2x + 2y - 6)$. Evaluate:

```python
implicit_plot(x+y-3,(x,-10,10),(y,-10,10)) +
implicit_plot(2*x+2*y-6,(x,-10,10),(y,-10,10), color="red")
```

(What happened? Where's the other graph???)

10. Find a parametrization by hand of $\mathcal{V}(x + y - 3, 2x + 2y - 6)$.

11. Now try `solve`. Evaluate:

```python
solve([x+y - 3, 2*x+2*y - 6],x,y)
```

What is the meaning of the answer Sage gave you?

12. What command would you type to get a nice picture of the twisted cubic $\mathcal{V}(y - x^2, z - x^3)$?

13. Find a description of the points on the twisted cubic. (What is the meaning of the answer Sage gives you?) Evaluate:

```python
solve([y-x^2,z-x^3],x,y,z)
```

14. What would you type to find a description of $\mathcal{V}(x^4 - zx, x^3 - yx)$?