Let $I = \langle x^2y^5, x^3y^4, x^4y^2 \rangle \subseteq \mathbb{R}[x, y].$

1. Draw a representation of the monomials contained in this ideal.

2. Find the set of monomials $x^\alpha$ such that $x^\alpha y^k$ is contained in $I$ for some $k \geq 0.$

3. Let $J$ be the ideal generated by these monomials. $J$ is an ideal in $\mathbb{R}[x]$ and thus has a single generator. Find the generator of $J.$

4. Let $m = \min\{k : x^2y^k \in J\}.$ Find $m.$

5. Let $J_0$ be ideal in $\mathbb{R}[x]$ generated by the set of monomials of the form $x^\beta$ where $x^\beta y^0 \in I.$ Find the generator of $J_0.$
6. Let \( J_1 \) be ideal in \( \mathbb{R}[x] \) generated by the set of monomials of the form \( x^\beta \) where \( x^\beta y^1 \in I \). Find the generator of \( J_1 \).

7. Let \( J_2 \) be ideal in \( \mathbb{R}[x] \) generated by the set of monomials of the form \( x^\beta \) where \( x^\beta y^2 \in I \). Find the generator of \( J_2 \).

8. Let \( J_3 \) be ideal in \( \mathbb{R}[x] \) generated by the set of monomials of the form \( x^\beta \) where \( x^\beta y^3 \in I \). Find the generator of \( J_3 \).

9. Let \( J_4 \) be ideal in \( \mathbb{R}[x] \) generated by the set of monomials of the form \( x^\beta \) where \( x^\beta y^4 \in I \). Find the generator of \( J_4 \).

10. Draw a representation of the monomials contained in the ideal generated by \( x^2 y^m \), and the generators of \( J_0, J_1, J_2, J_3 \) and \( J_4 \).