Let $I = \langle x^2y^5, x^3y^4, x^4y^2 \rangle \subseteq \mathbb{R}[x, y]$.

1. Draw a representation of the monomials contained in this ideal.

2. Find the set of monomials $x^\alpha$ such that $x^\alpha y^k$ is contained in $I$ for some $k \geq 0$.

3. Let $J$ be the ideal generated by these monomials. $J$ is an ideal in $\mathbb{R}[x]$ and thus has a single generator. Find the generator of $J$.

4. Let $m = \min\{k : x^2y^k \in J\}$. Find $m$. 
5. Let $J_0$ be ideal in $\mathbb{R}[x]$ generated by the set of monomials of the form $x^\beta$ where $x^\beta y^0 \in I$. Find the generator of $J_0$.

6. Let $J_1$ be ideal in $\mathbb{R}[x]$ generated by the set of monomials of the form $x^\beta$ where $x^\beta y^1 \in I$. Find the generator of $J_1$.

7. Let $J_2$ be ideal in $\mathbb{R}[x]$ generated by the set of monomials of the form $x^\beta$ where $x^\beta y^2 \in I$. Find the generator of $J_2$.

8. Let $J_3$ be ideal in $\mathbb{R}[x]$ generated by the set of monomials of the form $x^\beta$ where $x^\beta y^3 \in I$. Find the generator of $J_3$.

9. Let $J_4$ be ideal in $\mathbb{R}[x]$ generated by the set of monomials of the form $x^\beta$ where $x^\beta y^4 \in I$. Find the generator of $J_4$. 